Project STAIR

Teacher Reference Manual

Readiness
Individual
Algebra
Teaching of
Supporting
2.0

The original Project STAIR was supported by the Office of Special Education Programs (OSEP) under grant H326M170006. The project was housed at the University of Missouri, Southern Methodist University, and the University of Texas at Austin.

STAIR 2.0 is a federally funded research project that aims to accelerate pandemic recovery for middle school students with or at risk for disabilities in the area of mathematics. STAIR 2.0 is proudly funded by the Institute of Education Sciences (IES) under grant R324X220102 in efforts to promote research to accelerate pandemic recovery in Special Education. The project is housed at the University of Missouri, Southern Methodist University, and the University of Texas at Austin.

Principal Investigators:
Erica S. Lembke
Leanne Ketterlin- Geller
Sarah R Powell

Project Coordinator:
Stephanie Hopkins

Authors of this guide:
Kathleen Mason
Carolyn Conway
Danielle Lariviere
Leanne Ketterlin-Geller

Project STAIR 2.0 Team Members:

Sarah King Marcia Thomas Emily Singell Tara Atchinson-Green Heather Miller Destinee Boddie Gabriella DonofrioAnne Marie Light
Lizzie McCollum
Lois Ndugu
Lauren MacKenzie
Jess Mao
Charissa Richards

STAIR 2.0 Teacher Reference Manual

Welcome to the Project STAIR 2.0 takeaway resource book! We appreciate your participation in our project and would like to share best practices. The goal of this resource book is to provide easily accessible, concise, and evidence-based resources related to Project STAIR 2.0, including a review of implementing data-based individualization (DBI) to support middle-school algebra readiness. The resource book is divided into 6 units corresponding to the professional development modules, shown below.”

MAIN TOPICS INCLUDE

1) Algebra Readiness
2) Data-Based Individualization (DBI)
3) Assessments
4) Graphing and Decision-Making
5) Instruction
6) Intensification

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Introduction to Project STAIR

BACKGROUND ON PROJECT STAIR 1.0

As its name indicates, Project STAIR 2.0 is the second iteration of Project STAIR (Supporting Teaching of Algebra Individual Readiness). It is based off Project STAIR 1.0, which ran from 2018–2022. Project STAIR 1.0 was federally funded by the Office of Special Education Programs, which is part of the U.S. Department of Education. It was a four-year model demonstration project focused on supporting teachers at the middle school level who had students with mathematics difficulties. Knowing that passing algebra is a gatekeeper to later success in college and career, we sought to provide individual professional development and coaching support to improve middle school students' algebra reasoning. Much of this support centered on data-based individualization (DBI), a framework for assessing student progress and meeting students' individual instructional needs. Our research was conducted at three sites: Southern Methodist University, University of Missouri, and the University of Texas at Austin.

After participating in a full school year of STAIR 1.0, both teachers and students demonstrated improvement on assessments. Specifically, teachers demonstrated increased scores on understanding DBI, confidence using DBI, and frequency of DBI use. Students made significant growth from pre- to post-test on 2 of 3 pre-algebraic outcome measures. Results from STAIR 1.0 have been published in well-known educational peer reviewed journals, such as Studies in Educational Evaluation.

PROJECT STAIR 2.0 OVERVIEW

Using an established model developed as part of Project STAIR 1.0, the goal of STAIR 2.0 is to accelerate the pandemic recovery for middle-school students with or atrisk for mathematics disabilities. Project STAIR 2.0 centers on collaborative partnerships between coaches and teachers to provide timely, individualized mathematics support. Like Project STAIR 1.0, Project STAIR 2.0 is federally funded grant. Unlike Project STAIR 1.0, however, Project STAIR 2.0 is funded by a pandemic recovery grant through the Institute of Education Sciences (IES). IES recognized that the COVID-19 pandemic disrupted learning across the United States and created new challenges for educators and students. Therefore, they funded research to accelerate pandemic recovery in special education across the country over the next 3 years. Project STAIR 2.0 focuses on foundational mathematics in the middle-school grades that contributes to success in algebra. It provides professional development and coaching to teachers, utilizing practices tailored to teacher and student needs (i.e., DBI). The project runs from 2022–2025 across the same 3 sites as Project STAIR 1.0: Southern Methodist University, University of Missouri, and the University of Texas at Austin.

Introduction to Project STAIR

ALGEBRA READINESS AND DBI

The two primary focus areas within Project STAIR 2.0 are algebra readiness and DBI. These two focus areas go hand-in-hand, because DBI is used to build students' algebra readiness. Algebra readiness is comprised of two key components. The first is foundational skills, such as operating with rational numbers. The second is algebraic reasoning, such as solving for a variable in an equation. The remainder of this introductory unit will expand upon algebra readiness. The remainder of this introductory unit will address the rationale for using DBI and expand more upon algebra readiness.

WHY USE DBI?

Some students do not respond to research-based interventions. DBI provides a framework to individualize instruction for students who require intensive intervention. It is important to have a process an educator can follow when making meaningful instructional decisions for the students who need additional intervention or support.

Research in DBI has shown that when teachers use the DBI framework correctly, student achievement can improve (Powell et al., 2021). Regarding individualized instruction, research demonstrates that students who have intensive needs benefit from more practice and different instructional approaches to learn new information. In fact, these students require up to 10 to 30 times more practice than their peers do to acquire math skills (Fuchs, Fuchs, Powell, et al., 2008; Gersten et al., 2009). In other words, standard teaching techniques are simply not enough. Educators must organize their time to maximize students' learning opportunities, including focused instruction, and engaging, varied practice. Regarding assessment within DBI, educators must regularly evaluate their efforts to determine whether the current program is working. Importantly, a review by Stecker, Fuchs, and Fuchs (2005) noted that frequent progress monitoring with Curriculum-Based Measurement (CBM) is not enough, by itself, to improve student achievement. Instead, progress monitoring must be combined with systematic rules for using data to make decisions, analysis of students' skills, and guidance on making appropriate program modifications (Project STAIR, 2020). Each of these is a component of DBI.

Foundational Skills

FOUNDATIONAL SKILLS FOR ALGEBRA READINESS

Algebra readiness can be broken down into two major components: foundational skills related to algebra (e.g., rational number concepts, whole-number operations) and algebraic reasoning skills (e.g., interpreting expressions and equations, functions). This chapter is focused on foundational skills related to algebra readiness. For additional information on algebraic reasoning skills, see the next chapter.

The foundational skills that support algebra readiness are based in students' numerical proficiency. Specifically, these skills include (a) procedural fluency with whole-number operations, (b) conceptual understanding of rational-number systems, and (c) proficiency operating with rational numbers. This chapter will cover each of these foundational skills in detail. Figure 1 below can help you conceptualize and remember the role of these skills in supporting algebra readiness.

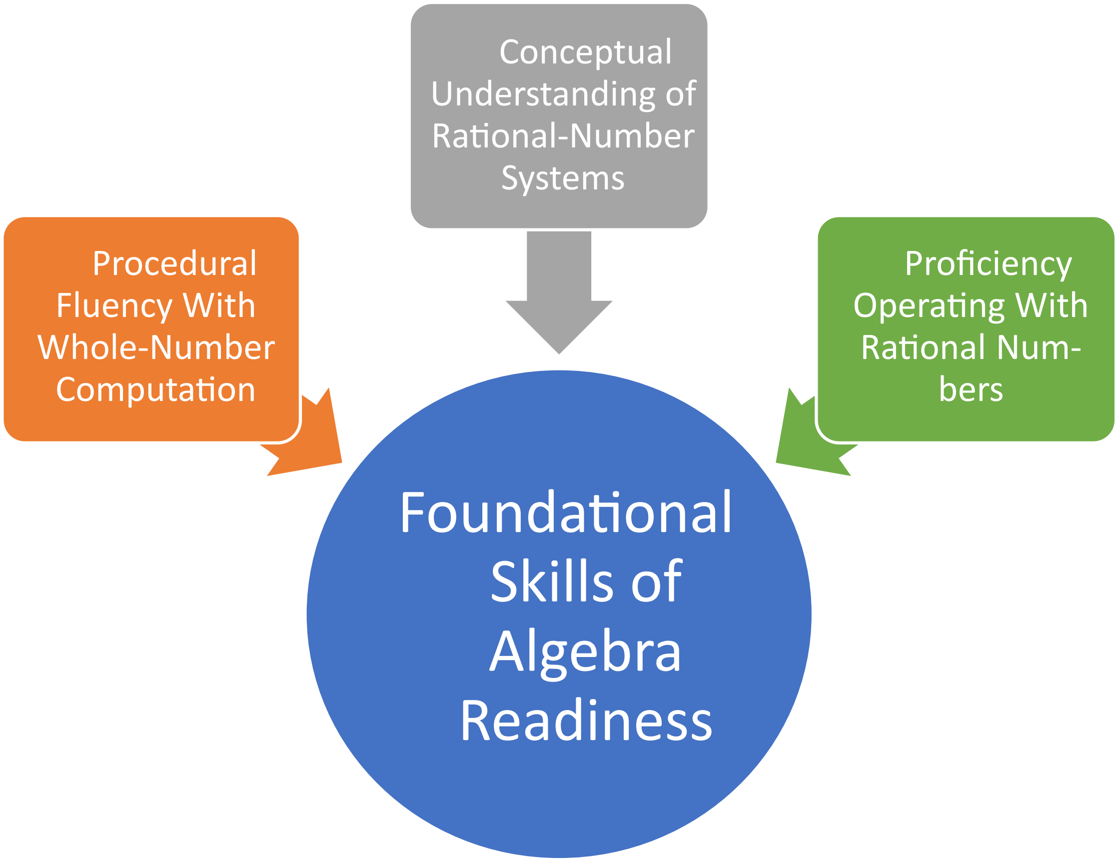

Figure 1. Foundational Skills of Algebra Readiness

Foundational Skills

PROCEDURAL FLUENCY WITH WHOLE-NUMBER COMPUTATION

In mathematics, the term “fluency” often refers to mastery of facts in addition, subtraction, multiplication, and division. While fact fluency is an important building block in mathematics proficiency, it is also important for students to develop fluency with more complex mathematics procedures (e.g., applying the distributive property or the order of operations). The National Research Council (2001) defines procedural fluency as the “knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them **flexibly**, **accurately** and **efficiently**.”

Students build procedural fluency with whole-number computation in a series of steps. First, they explore and discuss a number concept related to operating with whole numbers. For instance, what does it mean to multiply 4 by 13? Next, they develop informal reasoning strategies, such as skip counting (13, 26, 39, 52) or making 4 groups of 13 with manipulatives. Finally, they learn efficient procedures for solving problems involving whole-number operations, such as implementing the standard algorithm or partial product strategy (13 × 4 = 52). See Figure 2 for a visual of the progression to building procedural fluency with whole-number computation.

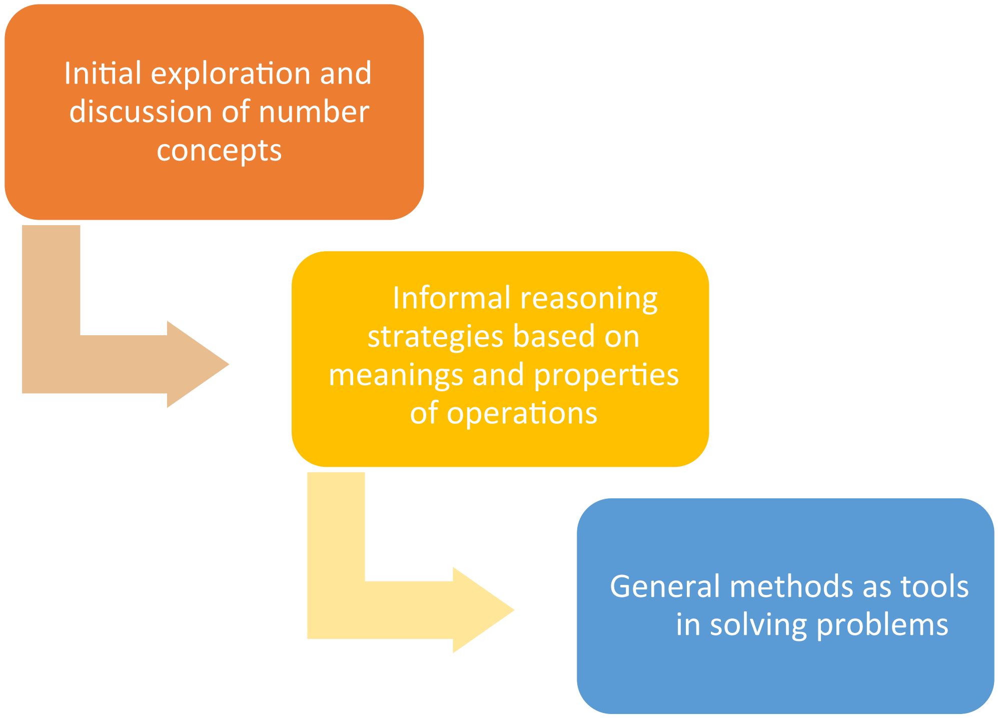

Figure 2. Progression of Building Procedural Fluency

Foundational Skills

PROFICIENCY OPERATING WITH RATIONAL NUMBERS

Whole-number computation is a fundamental component of algebra readiness. However, throughout late elementary school and middle school, students are also expected to perform operations with rational numbers. Rational numbers comprise the set of all numbers that can be represented as a fraction or a ratio of an integer to an integer. Fractions and decimals are common types of rational numbers used in algebra and algebra-readiness instruction. As students advance through middle school, they will work with negative rational numbers in increasing frequency. Rational numbers can be used to represent quantities in word problems (e.g., 3/4 of a pizza) and algebraic equations (e.g., $y = -0.4x + 1.82$). As students compute with rational numbers like fractions and decimals, they extend their procedural skills beyond the algorithms they have mastered with whole numbers.

CONCEPTUAL UNDERSTANDING OF RATIONAL-NUMBER SYSTEMS

While it is important to efficiently and accurately perform numerical operations, it is also critical to understand the conceptual underpinnings of those operations. Additionally, it is important to extend this conceptual understanding beyond whole numbers to rational numbers as well. Conceptual understanding in mathematics involves recognition of the intertwined relationships between mathematical concepts, patterns, and practices. This framework is used to (a) make connections between new and old knowledge, (b) recognize and fix procedural errors, and (c) choose appropriate strategies to solve unknown problems.

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Applied Example

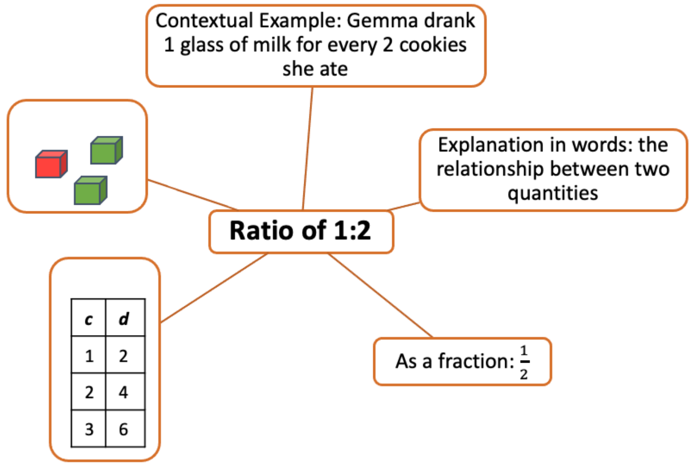


Figure 3. Visual Representation Diagram

Consider the range of visual representations in Figure 3. A student with conceptual understanding of the ratio can visualize it in multiple ways, recognizing that each visualization represents the same quantity but can be used in unique contexts. For instance, the ratio is used in a word problem about cookies and milk but is also used to generate a function table.

Algebraic Reasoning

ALGEBRAIC REASONING FOR ALGEBRA READINESS

Foundational skills (i.e., understanding and operating with whole and rational numbers) are critical for later proficiency in algebra. Another key component of algebra readiness is algebraic reasoning, which is the focus of this chapter. Algebraic reasoning centers on applying knowledge of concrete, arithmetic principles to abstract concepts (Pillay et al., 1998; Witzel, 2016). See Figure 4, which depicts students' progression from concrete to abstract mathematics and outlines how foundational skills help to build algebraic-reasoning ability.

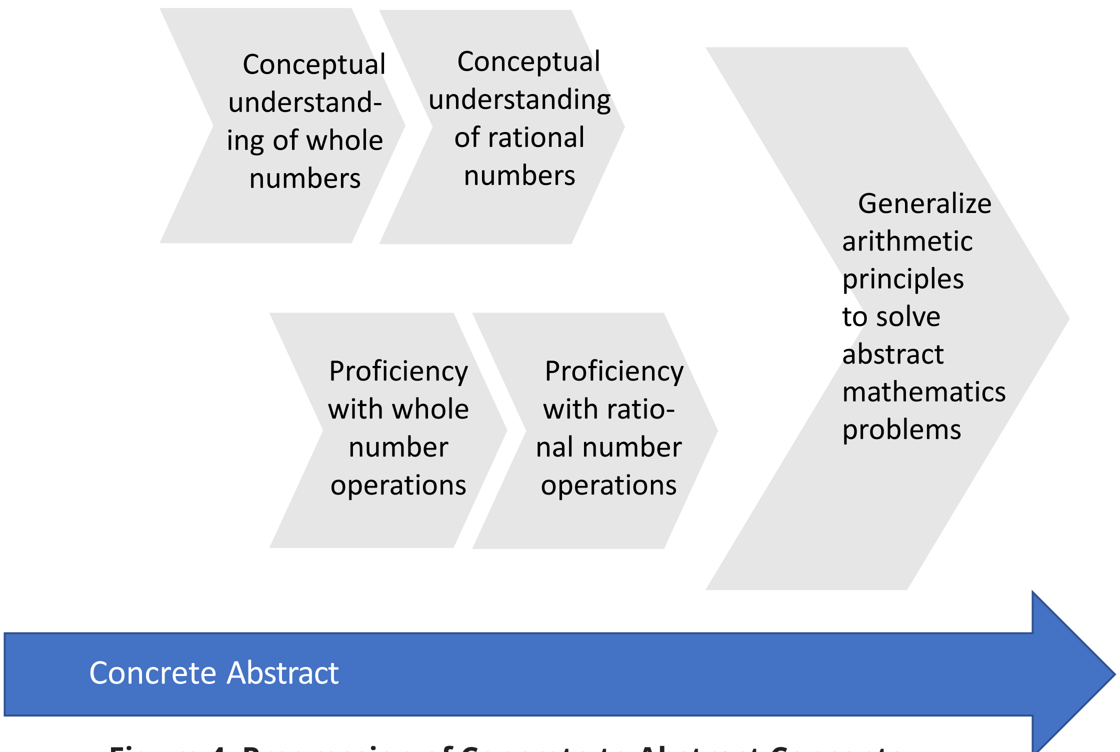


Figure 4. Progression of Concrete to Abstract Concepts

The last piece of the progression above, "General arithmetic principles to solve abstract mathematics problems," is the primary goal of algebraic reasoning. There are two major types of abstract problems to which middle-school students apply algebraic-reasoning skills: (a) solving for a variable in an equation and (b) modeling relationships with equations, graphs, and tables. The remainder of this chapter will discuss each of these problem types in additional detail.

Algebraic Reasoning

SOLVING EQUATIONS

A major strand of algebraic reasoning is solving for a variable in an equation, as students are regularly expected to demonstrate this skill with a range of equation types in Algebra 1. To prepare for the rigors of solving equations in Algebra 1, middle-school students work with algebraic expressions and learn to solve progressively more complex equations (CCSSM, 2010). First, students are exposed to relevant vocabulary terms and components of algebraic expressions. For instance, in the expression 3x+12, students learn to identify the 3 as a coefficient, the x as a variable, and the 12 as a constant. Students also practice applying mathematical properties to algebraic expressions. For instance, they learn about the equivalence of 5(x+3) and 5x+15.

As students prepare to apply their knowledge to solving equations, they utilize the fundamental principle of maintaining equal values on both sides of an equation. They apply this principle to solving progressively more complex equations, including those that require one step to solve, two steps to solve, combination of like terms, application of the distributive property, and variables on both sides of the equation. See Figure 5 for examples of each of these problem types.

Equation Elements	Example
One Step	−3□□ = 9
Two Steps	½□□ − 7 = 11
Combining Like Terms	10□□ − 11 + □□ + 13 = 24
Distributive Property	−6 □□ − 2 = 36
Variable on Both Sides of Equation	7 + 4□□ = 3□□ − 15

Figure 5. Examples of Equations

In addition to solving equations, students learn to solve inequalities such as 9x-28 ≥ -1. Finally, eighth grade students also learn to solve systems of equations for two variables, such as x+y=11 and 3x-y=21. Learning these skills helps students manipulate more complex expressions and equations when they reach Algebra 1 (e.g., polynomials).

Algebraic Reasoning

MODELING RELATIONSHIPS

Another key component of algebraic reasoning is modeling relationships between inputs and outputs. As middle-school students prepare for algebra, most exposure to this content centers on linear functions (CCSSM, 2010). Middle-school students learn to represent relationships such as linear functions in multiple ways: as an equation, in an input-output table, and on a coordinate plane. They use these skills to model real-life scenarios and word problems. For instance, consider the following scenario: A carnival has a $6.25 entrance fee and rides cost an additional $0.75 each. Students learn to identify $0.75 as the constant rate of change (i.e., the slope). They also learn to identify the $6.25 as the y-intercept. Students use this knowledge to represent the scenario in an equation, in a table, and on a coordinate plane. The equation in slope-intercept form would be $y=0.75x+6.25$, where y represents the total carnival cost and x represents the number of rides.

□□ (Number of rides)	□□ (Total cost)
1	$7.00
2	$7.75
3	$8.50

Figure 6. Input-Output Table

An input-output table would have several inputs representing different numbers of rides taken, as well as corresponding outputs representing total costs. See Figure 6 for an example of an input-output table for this scenario.

Finally, students learn to graph scenarios like this one on a coordinate plane. The x-axis represents the independent variable (number of rides) and the y-axis represents the dependent variable (total cost). See Figure 7 for a graphed representation of this scenario. As students gain experience modeling relationships in different ways, they improve their algebraic reasoning and boost their overall algebra readiness. When they encounter a range of relationships in Algebra 1 (e.g., linear, quadratic), they will have the tools to conceptualize these relationships flexibly and accurately.

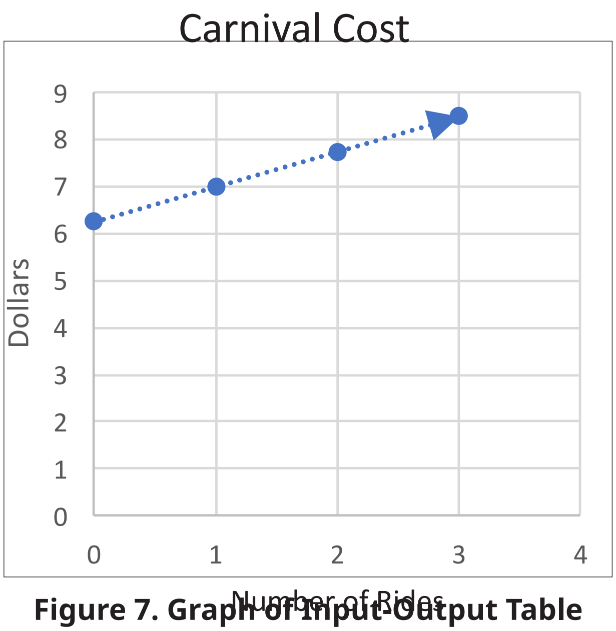


Figure 7. Graph of Input-Output Table

Defining DBI

WHAT IS DBI FOR MIDDLE SCHOOL MATHEMATICS?

Data-based individualization (DBI) for middle-school mathematics is a systematic, data-based approach to individualizing instruction. The primary goal of such instruction is to support students' algebraic reasoning, thereby preparing them for the rigors of algebra coursework in late middle school or high school. Figure 8 displays the DBI framework, which was developed by the National Center for Intensive Intervention (NCII).

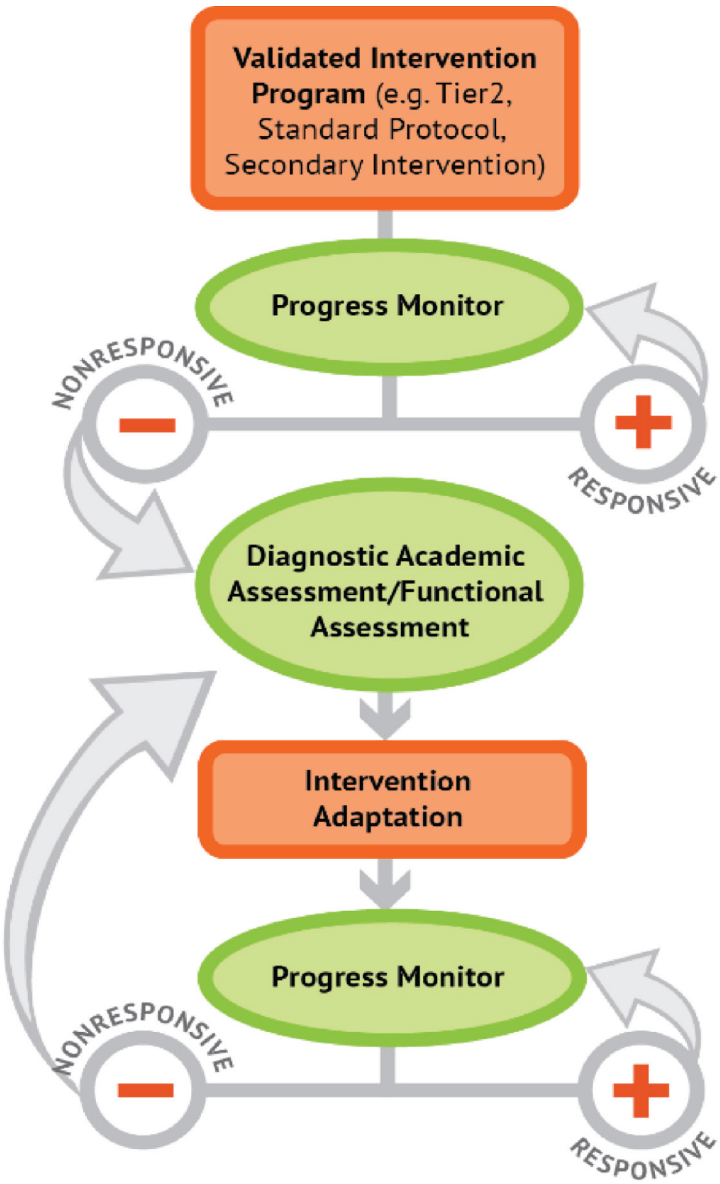


Figure 8. NCII's DBI Framework (NCII, 2020)

Key Components of DBI

WHAT ARE THE MAIN ELEMENTS OF DBI?

Data-based individualization (DBI) integrates instructional design principles and assessments to create individualized, responsive intervention for students with persistent learning needs. DBI serves as the overarching approach for addressing individual student needs when learning pre-algebraic concepts. DBI provides the context and rationale for for integrating formative assessment data with teachers' decisions about the selection and use of evidence-based instructional practices (Project STAIR, 2020).

The three main characteristics of DBI are that it is (a) systematic, (b) data-driven, and (c) individualized. When utilizing DBI, teachers systematically collect student data and implement evidence-based interventions. The second characteristic encourages reliance on data for instructional decision making in classrooms. This data-driven approach supports teachers to make objective changes when considering students' response to instruction. The third characteristic of DBI is to individualize instruction for students who are most at risk for encountering difficulties with the targeted concepts (Project STAIR, 2020).

Key Points about the DBI Framework (NCII, 2020)

- DBI is a validated process, not a single intervention program or strategy.
- DBI is an ongoing process in which intervention and assessment are linked and used to adjust a student's academic or behavior program over time. DBI is not a one-time fix.
- DBI is often domain-specific, meaning that a student may receive DBI in one domain (e.g., mathematics or reading) or even on one component of that domain (e.g., computation or fluency) while receiving core or supplemental instruction in other domains. DBI can also be implemented in multiple domains at the same time, responding to the learning needs of the student.
- Students with the most intensive needs will likely require DBI over a sustained period of time. Decisions about if and when to reduce the intensity and individualization of the intervention must take into account the student's responsiveness, as well as the breadth and nature of skill.

Evidence Behind DBI

WHAT EVIDENCE SUPPORTS DBI?

Data-based individualization (DBI) is a framework for intensifying intervention in which student-level formative assessment data are systematically used to determine when and how a student's intervention program should be modified (NCII, 2013). Research demonstrates that students who have intensive needs benefit from more practice and different instructional approaches to learn new information. In fact, these students require up to 10 to 30 times more practice than their peers do to acquire new math skills (Fuchs, Fuchs, Powell, et al., 2008; Gersten et al., 2009). In other words, standard teaching techniques are simply not enough. Educators must organize their time to maximize students' learning opportunities, including focusing instruction and providing engaging, varied practice. Furthermore, they must regularly evaluate their efforts to determine whether the current program is working. Importantly, a review by Stecker, Fuchs, and Fuchs (2005) noted that frequent progress monitoring with Curriculum-Based Measurement (CBM) is not enough, by itself, to improve student achievement. Instead, progress monitoring must be combined with systematic rules for using data to analyze students' skills and make appropriate program modifications (Project STAIR, 2020).

DBI is often implemented within a multi-tiered systems of support (MTSS) framework, such as RTI, to support students for whom core instruction (i.e., Tier 1) and secondary intervention (i.e., Tier 2) have been insufficient to facilitate adequate academic or behavioral progress. DBI may supplement or supplant Tier 1 and 2 supports depending on student need and may be applied to a specific skill area. That is, a student may develop proficiency in aspects of measurement and geometry through core instruction and achieve grade-level computational fluency with secondary intervention, but that student may require DBI to improve specific algebra-readiness skills. Alternatively, a student with global mathematics difficulties may require DBI in all areas (Project STAIR, 2020).

WHY IMPLEMENT DBI?

Some students do not respond adequately to research-based interventions. In these cases, DBI provides a framework to individualize instruction. It is important to have a process an educator can follow to making meaningful instructional decisions for students who need additional intervention or support. Research in DBI has shown that when teachers use the DBI framework correctly, student achievement can improve. It is important to have a process an educator can follow when making meaningful instructional decisions for students who need additional intervention or support.

Key Assessment Types and Uses

ASSESSMENTS WITHIN A DBI FRAMEWORK

Utilizing formative assessment data has been considered by many to be a key aspect of effective instruction. For students, specifically those experiencing mathematics difficulties or disabilities, formative assessment data provides feedback regarding their performance or effort. Ongoing assessment of students' progress in mathematics can therefore help teachers measure the pulse and rhythm of students' growth and also help them fine-tune instruction to meet students' needs.

Within the DBI process, essential sources of data come from administering three different types of assessments: universal screening, progress monitoring, and diagnostic assessment. Each assessment and the information gathered from it serves a different purpose in the DBI process.

The purpose of universal screening is to identify students who may be at risk for poor academic outcomes, including students who require intensive intervention. Progress monitoring is administered by teachers to monitor students' progress toward their goal and make timely and ongoing decisions about students' response to the current intervention. Last, diagnostic assessments are administered to identify students' persistent misconceptions and errors.

In Project STAIR, data from universal screeners such as STAR (Renaissance Learning, 2011) was used to establish students' present level of performance and select students for whom DBI was appropriate. For progress monitoring, Algebra Readiness Progress Monitoring (ARPM; Ketterlin-Geller et al., 2015) was used for repeated measurements of students' mathematics performance on three key algebraic readiness constructs. Last, Diagnosis Online Math Achievement (DOMA; Let's Go Learn, Inc., 2019) was used to identify students' level of mastery in algebra-readiness topics and pinpoint possible sources of misconceptions and errors.

In the following section, the purpose and evidence supporting the use of each type of assessment will be discussed. Also, you can find resources to support the selection and evaluation of screening, progress monitoring, and diagnostic assessments.

WHAT IS A UNIVERSAL SCREENER?

A universal screening assessment is a type of assessment that is characterized by providing quick, low-cost, repeatable testing of age-appropriate critical skills. Universal screening is administered to identify students who may be at risk for poor learning outcomes and who need additional assessment (i.e., progress monitoring) and support.

Universal Screening

Universal screening data provide information on the effectiveness of the core instruction and curriculum. Universal screening is considered a key feature of early intervention and an important first step for identification of students at-risk for learning difficulties or learning disabilities.

EVIDENCE FOR UNIVERSAL SCREENING

There have been a number of articles and sources discussing the use of universal screeners (Glover et al., 2007; Fuchs et al., 2011; Catts et al., 2009). For example, the IES practice guide (2009) introduces recommendations to systematically use universal screening to screen all students to determine which students have mathematics difficulties and require research-based interventions. This recommendation is based on a series of high-quality correlational studies with replicated findings that show the ability of measures to predict performance in mathematics one to two years after administration.

HOW TO SELECT UNIVERSAL SCREENING MEASURES

Schools should evaluate and select screening measures that are efficient and technically rigorous in (a) Classification Accuracy, (b) Technical Standards (i.e., reliability, validity), and (c) Usability Features.

The basic function of a screening measure is to identify who may need supplemental instruction to reach the curricular expectations. Therefore, these tests must meet empirical psychometric qualities of reliability and validity (e.g., content validity, predictive validity, etc.).

USING UNIVERSAL SCREENING DATA TO SELECT STUDENTS FOR DBI

Universal screening results should be used to answer the following questions:

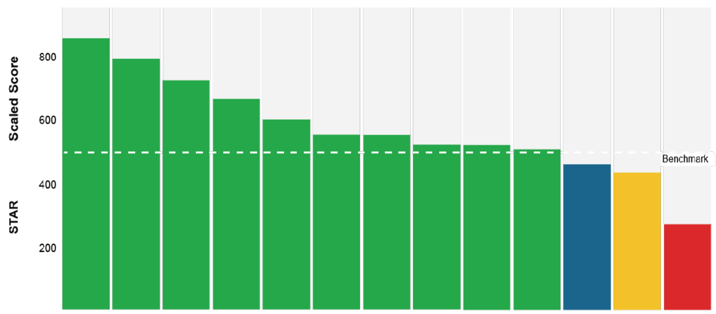
» **Which, if any, students may need supplemental instruction to reach the curriculum expectations?**

» **What degree of intensity of intervention is needed?**

Universal Screening

DECISION-MAKING FROM UNIVERSAL SCREENING RESULTS

Figure 9 below is a sample score report from the STAR assessment by Renaissance Learning. This report can be used to identify students as candidates for DBI. The report shows that as the scaled score goes down, the student's risk status goes up. question on p. 17 can be answered based on the STAR assessment screening data.



Categories / Levels	Benchmark: Scaled Score	Benchmark: Percentile Rank	Students: Number	Students: Percent
At/Above Benchmark				
At/Above Benchmark	At/Above 514 SS	At/Above 40 PR	10	77%
			10	77%
Below Benchmark				
On Watch	Below 514 SS	Below 40 PR	1	8%
Intervention	Below 444 SS	Below 25 PR	1	8%
Urgent Intervention	Below 337 SS	Below 10 PR	1	8%

Which (if any) students are at-risk or underperforming?

» **Green** Students at or above benchmark are meeting or exceeding grade-level expectations.

» **Blue** Students performing below the 40th percentile and above the 25th percentile are not at-risk or underperforming.

» **Yellow** Students performing below the 25th percentile and above the 10th percentile are at-risk and in need of supplemental intervention.

» **Red** Students performing below 10th percentile are under-performing and in need of immediate intervention.

Figure 9. Sample Score Report of a STAR Assessment

What degree of intensity of intervention is needed?

» **Yellow** The students in the yellow likely need Tier-2 intervention and may need more intensive, individualized intervention.

» **Red** The students in the red are candidates for intensive, individualized intervention.

Once students have been screened and identified as at-risk or underperforming, a Tier 2 intervention is administered according to the school's plan. The next step is to measure the progress students are making with the Individualized Education Program (IEP) goals.

Progress Monitoring

WHAT IS PROGRESS MONITORING?

The purpose of progress monitoring is to assess students' response to primary, secondary, and tertiary instruction. Through this process, teachers can identify students who are not making adequate progress so that instructional changes can be made. A progress-monitoring assessment is a valid and efficient tool for gauging the effectiveness of instruction, determining whether instructional modifications are necessary, and providing important information for eventual classification and placement decisions.

Progress monitoring allows practitioners to:

- estimate rates of improvement
- identify students who are not demonstrating adequate progress
- compare the efficacy of different forms of instruction to design more effective, individualized instruction.

HOW TO SELECT PROGRESS-MONITORING MEASURES

Schools should evaluate and select progress monitoring measures that are efficient and technically rigorous in (a) Performance Level Standards, (b) Growth Standards, and (c) Usability.

Progress-monitoring assessments should (a) be quick and easy to administer; (b) have multiple parallel forms with the same difficulty, format, and content; and (c) follow standardized administration and scoring with the same timing, setting, and scoring rules.

PROGRESS MONITORING DATA EVALUATION

Progress-monitoring results should be used to answer the following questions.

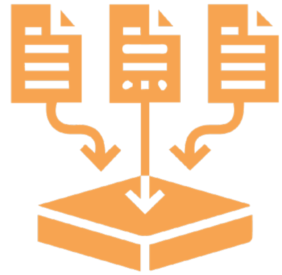
» **Is the student making adequate progress toward their goals?**
» **Is the intervention effectively meeting the student's needs?**

There are five steps to progress monitoring: 1. Gather baseline data; 2. Set performance goals; 3. Implement the intervention; 4. Monitor routinely; 5. Evaluate the effectiveness of the intervention.

Progress Monitoring

01 Gather Baseline Data

Baseline data enables us to evaluate student performance prior to any instructional changes. When you gather baseline data, be sure to collect at least three data points. Student performance may vary based on many factors, so we recommend taking the median of three scores collected within one to two weeks to have an accurate picture of their starting score.



02 Set Performance Goals

Based on the baseline data, you should set a performance goal to the level you expect your student to improve. The goal should be ambitious but reasonable. For example, in the Progress Monitoring Chart, Figure 10 on page 23, the green line is the student's performance goal (i.e., aim line). It means that the student's performance is expected to improve up to score of 90 after 15 weeks.

03 Implement Intervention

Once you've set a performance goal, then you are good to start implementing your intervention. Remember that intervention you're giving should be systematically planned based on student's assessment data and it should be evidence-based.

04 Progress Monitor Routinely

It is important to measure student's progress on a regular basis. In the Progress Monitoring Chart, refer to Figure 10 on page 23, the teacher implemented the intervention and administered progress monitoring measures once a week. Routine progress monitoring provides valuable insight into the effectiveness of the intervention as well as the student's responsiveness to the intervention.

05 Evaluate Effectiveness

There are decision-making rules for evaluating student progress and intervention effectiveness by comparing the trend line to the goal line. Review the graph weekly following at least 4 initial weekly data points. For example, if teacher planned to administer progress monitoring measure once per week, review the graph every week. Further information for decision-making will be provided in the Graphing' and 'Decision-Making' sections.

Diagnostic Assessment

WHAT IS A DIAGNOSTIC ASSESSMENT?

A diagnostic assessment is a form of assessment where teachers can evaluate students' mastery of relevant prior knowledge and skills on certain topics as well as preconceptions or misconceptions about the material before and during their instruction. A diagnostic assessment helps teachers fine-tune instruction to meet the needs of their students and provide students with information regarding their performance.

EVIDENCE FOR DIAGNOSTIC ASSESSMENTS

According to Gersten's (2009) meta-analysis on instructional approaches that enhance mathematics proficiency of students with learning disabilities, using formative assessment data to address students' instructional needs had a small to medium effect size.

USING DIAGNOSTIC ASSESSMENT DATA TO PLAN INTERVENTION

A diagnostic assessment results should be used to answer the following ques-

- **» Why is the student underperforming?**
    - **» What are the student's correct conceptualizations or understandings of the content?**
    - **» What are the student's persistent misconceptions and errors?**
- **» What content or instruction design features should be included in the inter vention for this student?**

Teachers can use diagnostic assessment to gain insight into specific students' levels of knowledge, skills, and understanding before and during units and/or lessons. Diagnostic assessment helps teachers think about the content that they need to work on with their students. To determine which skills to target with students, consider the following steps.

For each of your students, identify prior knowledge from which instruction should build and areas that need improvement. Prioritize the content based on:

- current scope and sequence
- importance of content for future instruction
- significance of gaps.

The Graphing Process

HOW DOES GRAPHING FIT WITHIN THE DBI FRAMEWORK?

You'll recall that DBI has three main elements: it is (a) systematic, (b) data-driven, and (c) individualized. Collecting progress-monitoring data is systematic and frequent so that data can be used reliably to inform the individualization of instruction.

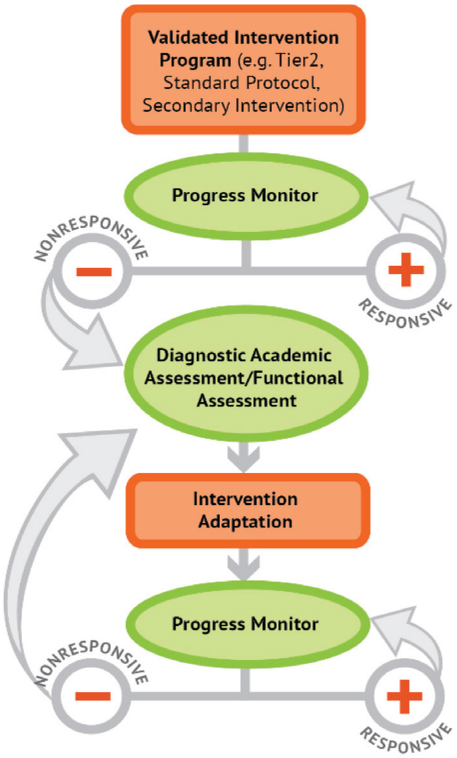


Figure 8. NCII's DBI Framework (NCII, 2020)

Progress-monitoring data best supports instructional decision making when it is graphed regularly. This allows for a visual of student progress relative to the goals you've set for quick decision making. The progress-monitoring graph will allow you to easily:

- set reasonable and ambitious goals
- monitor the appropriateness of the student's goal
- judge the adequacy of the student's progress
- determine the effectiveness of the student's mathematics instructional program
- use decision rules to make changes to the student's instructional program when needed.

GRAPHING PROGRESS-MONITORING DATA FOR DECISION-MAKING

This allows you to constantly have your finger on the pulse of your student's progress in response to the instructional adaptations you've made. The Graphing Process follows repeated iteration of establishing a baseline, setting a goal, and monitoring progress. Figure 10 is an example of a progress-monitoring chart.

The Graphing Process

Key Terms

Baseline	Scores that represent median student performance prior to initiating intervention
Slope	Growth rate of student progress
Trend-line	A line which shows the direction of the student's overall growth during the intervention period
Goal Line	A line which connects the baseline score to the score expected at the end of the intervention.

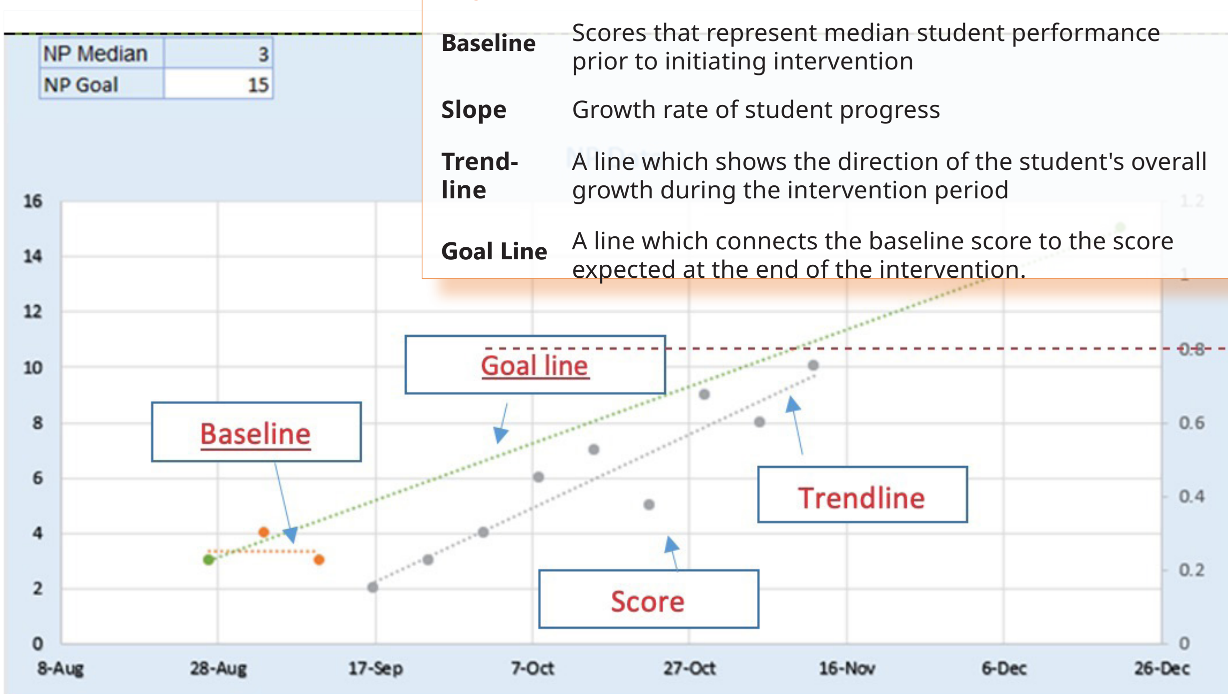


Figure 10. Progress Monitoring Chart

GRAPHING PROCESS

1. BENCHMARK

The first step to setting up a graph is to enter the three benchmark scores you collected upon initiating progress monitoring. This establishes a baseline of where your student is performing prior to any instructional changes.

2. CALCULATE

The next step is to set a reasonable but ambitious goal. By the end of the intervention period, how much progress do you expect the student to make? We recommend calculating a goal based on growth rate (slope). See equation below.

3. GRAPH

The next step is to enter the student's score from the progress-monitoring measure each week as it is collected. Each time you enter a new score, the graph will update the trendline. This shows the general trend of student progress and can be compared to the goal line for decision making.

Calculating a Goal

Goal = Baseline Score + Growth rate * numBer of Weeks

Decision-Making

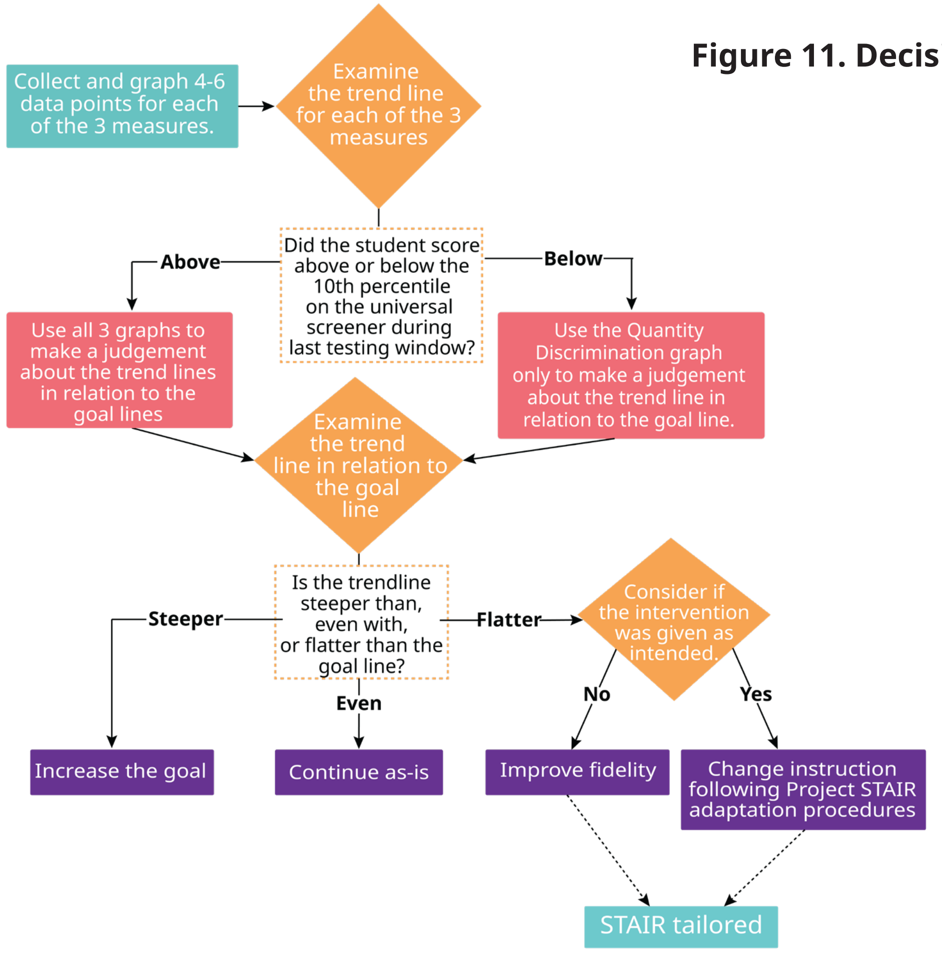
DECISION-MAKING FLOWCHART

As you are reviewing your graphed data to make decisions about instruction adaptations or individualization, you should apply consistent decision rules. These decision rules, based on the following questions, are evidence-based to best support your data-based decision making.

Is the trend line:

- **above the goal line (i.e., is the slope of the trend line steeper than the slope of the goal line)?**
- **even with the goal line (i.e., is the slope of the trend line the same as the slope of the goal line)?**
- **below the goal line (i.e., is the slope of the trend line flatter than the slope of the goal line)?**

Figure 11. Decision-Making Flowchart



Goal-Setting

HOW TO USE THE DECISION-MAKING FLOW CHART

The first step in using the decision-making flowchart is to determine if it is an appropriate time to make a decision. If you've been implementing something new, you cannot be sure if it is the cause of change in data, or a lack of change in the data, until an appropriate amount of time has passed.

Once you've identified the graph you are looking at, you should examine the relationship of the trendline to the goal line. If the trendline is:

- steeper than the goal line, this indicates that the student is on track to
- exceed the goal. You can consider increasing the goal.
- even with the goal line, this indicates that the student is on track to meet
- their goal. You should continue as is and observe the trendline weekly.
- flatter than the goal line, this indicates that the student is not on track to meet their goal.

Fidelity of Intervention

In the last instance, you will want to consider the fidelity of implementation. Some considerations:

» **Was the intervention administered as intended?**

» **Was the student present for all of the intervention?**

If the answer to either of these questions is no, the first step is to address fidelity and revisit the graph after collecting 4-6 data points. If fidelity is not a concern, you should consider a change in instruction.

SETTING GOALS

It is important to set a reasonable and ambitious goal over an easy goal. So what is an ambitious goal? This can be determined using the following equation, using a slope of 1 to calculate the goal:

Goal = [[Number of weeks of the intervention] x Target Growth Rate] + [Baseline Score]

Take the following example of an intervention:
The intervention is **8 weeks long**; the **target growth rate is .50**, determined from the intervention ROI tables; the **baseline score was 19**. The calculation would be:

Goal = [8] x [.50] + [19] = 23

Introduction to Instruction

INTRODUCTION TO INSTRUCTION

This section will provide an overview of evidence-based strategies to incorporate into algebra-readiness instruction. The following instructional strategies will be covered: explicit instruction, multiple representations, mathematics language, fluency building, word problem-solving, and graphic organizers. Explicit instruction involves modeling, opportunities for practice, and supporting practices such as providing feedback. Multiple representations include concrete, pictorial, and abstract versions of mathematics concepts. Mathematics language incorporates specific terminology and language structures to communicate mathematics concepts using precise language. Fluency building refers to both fact recall and accurate implementation of mathematics procedures. Word problem-solving includes the use of attack strategies and schema instruction. Graphic organizers refer to visual diagrams that support mathematics learning. Additional details on each of these instructional strategies will be provided in the remainder of this unit.

These strategies have been selected because they have demonstrated efficacy in mathematics interventions for students with mathematics difficulty, including middle-school students preparing for the rigors of algebra (Gersten et al., 2009; Kroeger & Kouche, 2006; Powell et al., 2021). Keep in mind that these strategies need not be used in isolation. Many interventions incorporate several of these strategies simultaneously (Powell et al., 2021).

EXPLICIT INSTRUCTION

Explicit, systematic instruction is essential when providing mathematics intervention (Fuchs et al., 2021). In reviews of mathematics intervention studies, explicit instruction is often noted as a critical component of effective interventions (Jitendra et al., 2018; Stevens et al., 2018).

Researchers have used explicit instruction to teach a range of mathematics content, including operations (Milton et al., 2019; Parker et al., 2019), fractions (Bouck et al., 2017; Dyson et al., 2020), word-problem solving (Bottge et al., 2014; Jitendra & Star, 2012), algebra (Bouck et al., 2019; Bryant et al., 2020), and mathematics writing (Hughes & Lee, 2020).

Explicit Instruction

Explicit instruction incorporates three key components: modeling, practice, and supports. Modeling includes clear step-by-step explanations with planned examples. Practice includes guided practice (i.e., practice with the teacher) and independent practice. During modeling and practice, teachers should use supports, such as asking low and high-level questions, eliciting student responses, providing immediate corrective feedback, and maintaining a brisk pace. See Figure 12 for a visual to help you remember these components and how they fit together.

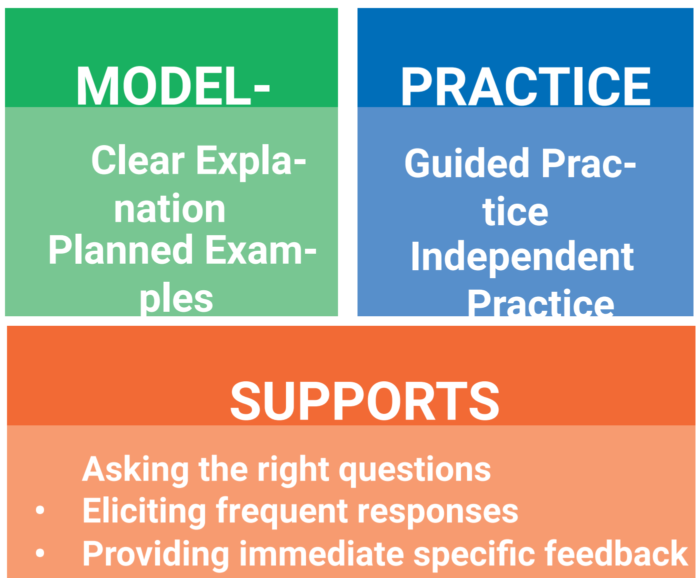


Figure 12. Explicit Instruction Visual

MODELING

When modeling, start with a clear explanation using precise mathematics language. Explain why the content is important. For example, say, "Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends." This will help students start to connect the content and real life. Precise language includes the formal language of mathematics. For example, say "numerator" instead of "top number" or "product" instead of "answer."

While modeling, model the steps to solve a problem. Involve your students by asking questions and giving them opportunities to respond. Modeling should feel like a dialogue between you and your students.

Explicit Instruction

Plan the examples you choose for your interventions. For example, when modeling a division problem, consider the different ways to show division and how you want to represent the problems. Include non-examples to help students understand when to apply the strategy modeled. See Figure 13 below for how to implement modeling with your students.

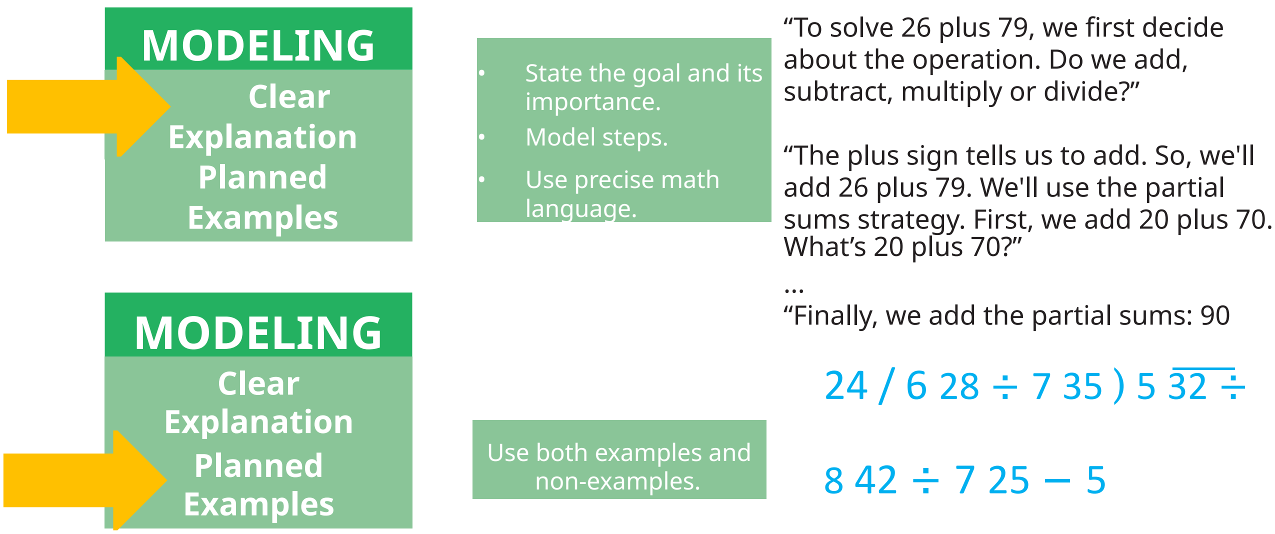


Figure 13. How to Implement Modeling

PRACTICE

Practice includes both guided and independent practice opportunities for students. During guided practice, provide scaffolding as needed while practicing together with the student. During independent practice, students practice independently while you provide feedback. Ensure that students can complete problems on their own before they begin independent practice. See Figure 14 for a diagram of guided and independent practice that includes some sample language.

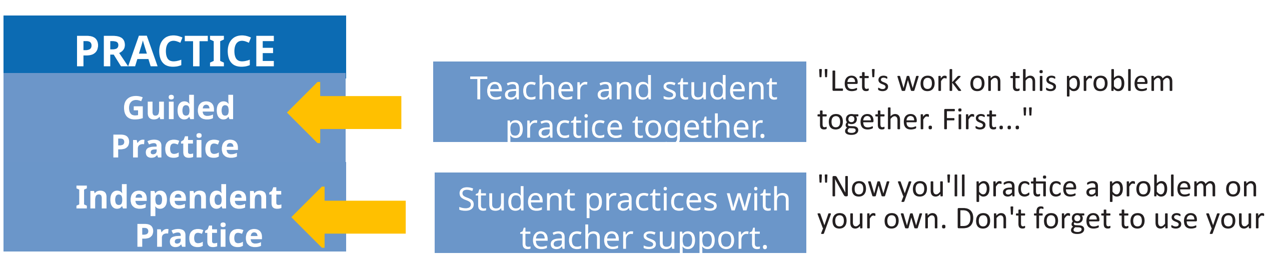


Figure 14. Diagram of Practice

Explicit Instruction

SUPPORTS

During modeling and practice, it is important to provide the following supports: asking the right questions, eliciting frequent responses, providing immediate, specific feedback, and maintaining a brisk pace. To ask the right questions, ask a mix of low-level questions (to check for understanding) and high-level questions (to learn what students understand about different concepts and procedures). Give students opportunities to respond every 30 to 60 seconds to elicit frequent responses. When providing feedback, offer specific praise for student performance with the mathematics content. Provide corrective feedback promptly when needed. Finally, maintain a brisk pace by being prepared, organized, and ready to teach to maximize learning time. See Figure 15 below for a diagram depicting the implementation of each of these supports.

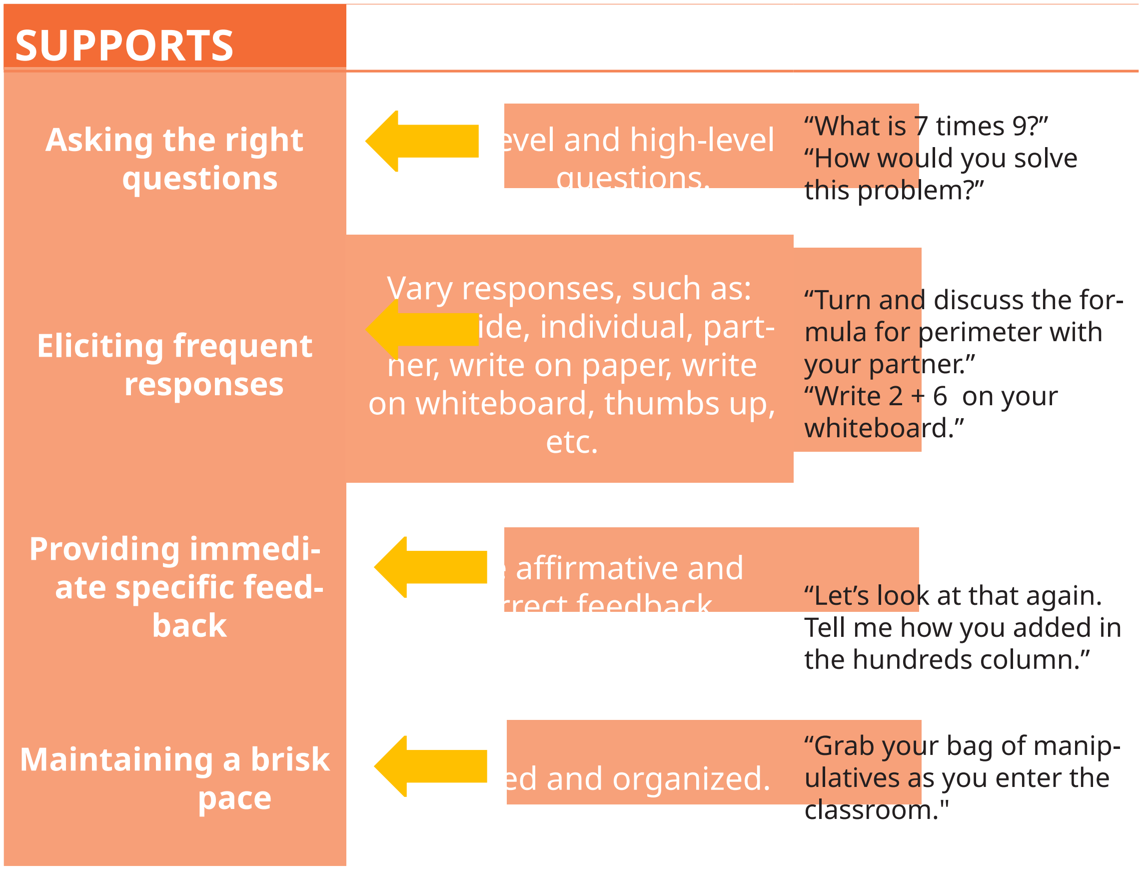


Figure 15. How to Implement Supports

Multiple Representations

WHAT ARE MULTIPLE REPRESENTATIONS?

Teachers should use multiple representations to help students understand mathematics concepts and procedures. Multiple representations include concrete representations. Students can use hands-on tools and manipulatives to show mathematics concepts. Multiple representations also include pictorial representations. These might be pictures, graphs, or organizers printed or drawn on a two-dimensional surface. Pictorial representations also include virtual manipulatives accessed via technology. Finally, multiple representations include abstract representations of mathematics presented with numerals, symbols, and words. See Figure 16 for an image of how these representations fit together conceptually.

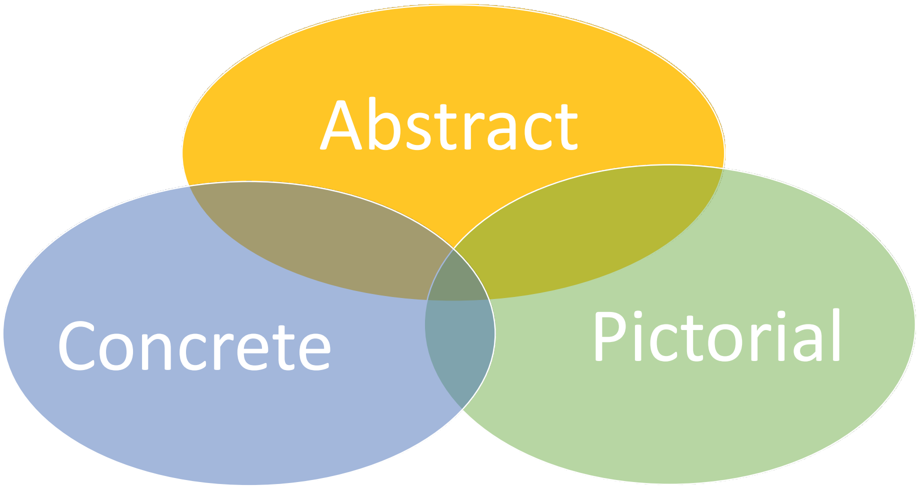


Figure 16. Multiple Representation Conceptualization

As you can see, these representations fit together into a framework rather than a strict sequence. You can use multiple types of representations in a single lesson. Additionally, you can use one representation (e.g., fraction bars) and return to it after introducing other representations (e.g., pictorial and symbolic representations of fractions).

When students use multiple representations, their mathematics performance increases (Bouck & Park, 2018). This is true for hands-on tools (Namkung & Bricko, 2021), virtual manipulatives (Bouck et al., 2018), and graphic organizers. Multiple representations have been used to increase student understanding of operations (Bennett & Rule, 2005), fractions (Bouck et al., 2020), word-problem solving (Xin et al., 2020), geometry (Strickland & Maccini, 2012), and algebra (Scheuermann et al., 2009).

Multiple Representations

CONCRETE REPRESENTATIONS

Concrete representations include three-dimensional objects that help make mathematics concepts more tangible for students. They are used across a range of mathematics domains, such as fractions, algebra, and geometry. Some examples include dice, base-ten blocks, and geoboards. Figure 17 shows examples of concrete manipulatives. Note that many of these manipulatives can be used across multiple mathematics domains.

Operations	Whole Numbers	Fractions	Algebra	Geometry

Figure 17. Examples of Concrete Representations

PICTORIAL/VISUAL REPRESENTATIONS

Visual representations include two-dimensional objects and virtual manipulatives. Like concrete manipulatives, visual representations are used across multiple mathematics domains. Some examples include digital number lines and coordinate planes. See Figure 18 for examples that are useful in particular mathematics domains.

Multiple Representations

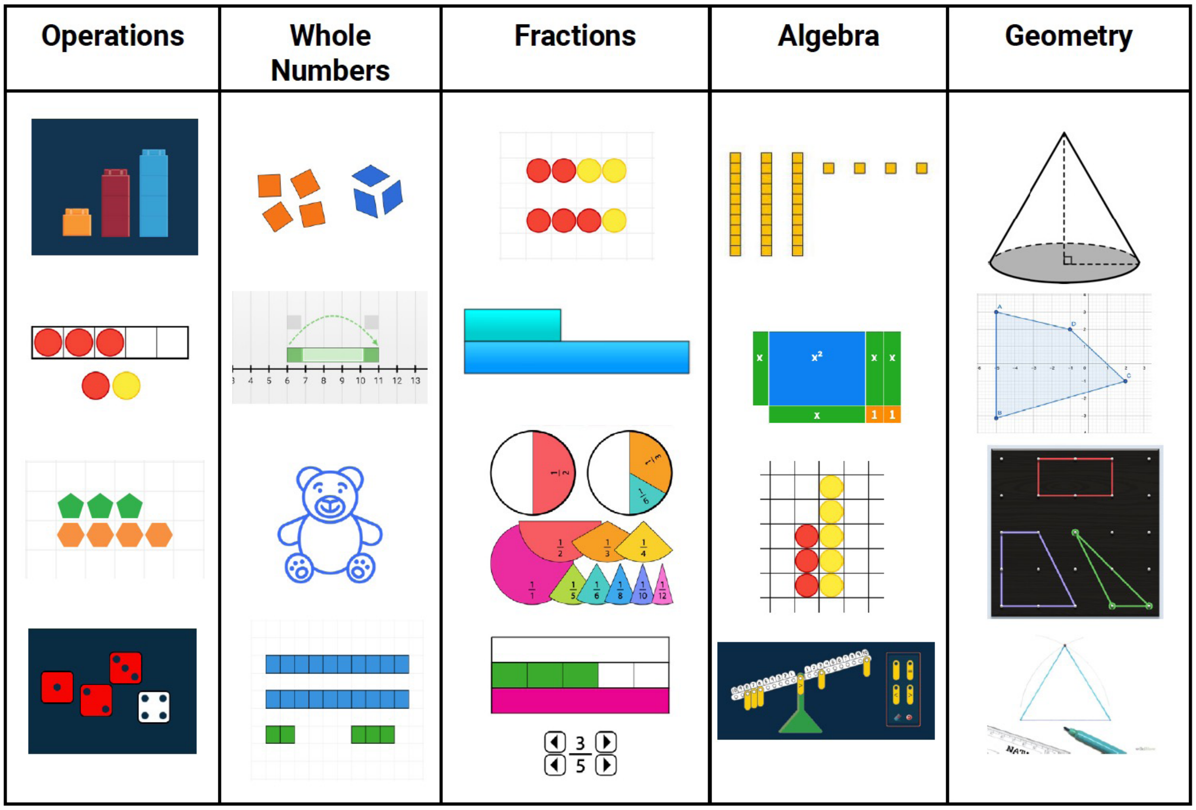


Figure 18. Examples of Pictorial/Visual Representations

ABSTRACT REPRESENTATIONS

Abstract representations include symbols and numerals. It is best to present abstract representations simultaneously with concrete or pictorial representations. See Figure 19 below for some examples of abstract representations of mathematics content.

$$78 = 7 \text{ tens and } 8$$

$$4 + 9 = 13$$

$$\begin{array}{r} 4{,}156 \\ \underline{+ \quad 789} \end{array}$$

$$x - 6 = 9$$

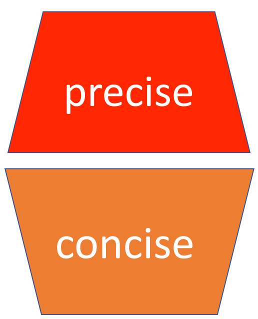
Figure 19. Examples of Abstract Representations

Mathematics Language

MATHEMATICS LANGUAGE

Precise mathematics language includes the formal mathematics terms that accurately portray a mathematics concept or procedure. Teachers should use precise and concise mathematics language to introduce terms (e.g., numerator rather than top number). When teachers use concise mathematics language, they use short, accurate, student-friendly definitions to discuss mathematics concepts.

Research indicates that teachers and students should have multiple opportunities to access and use precise and concise mathematics language (Powell et al., 2019). Teachers who use informal language or casual terms (e.g., box instead of cube) are setting students up for later confusion when they are assessed using formal language or move to another classroom where a different term is used. See Figure 20 for examples of precise and concise mathematics language commonly used in middle school.



"The denominator is the number of equal parts that make the whole."

"To show the fraction, look at the denominator. A denominator of 5 means I need to break the whole into 5 equal parts."

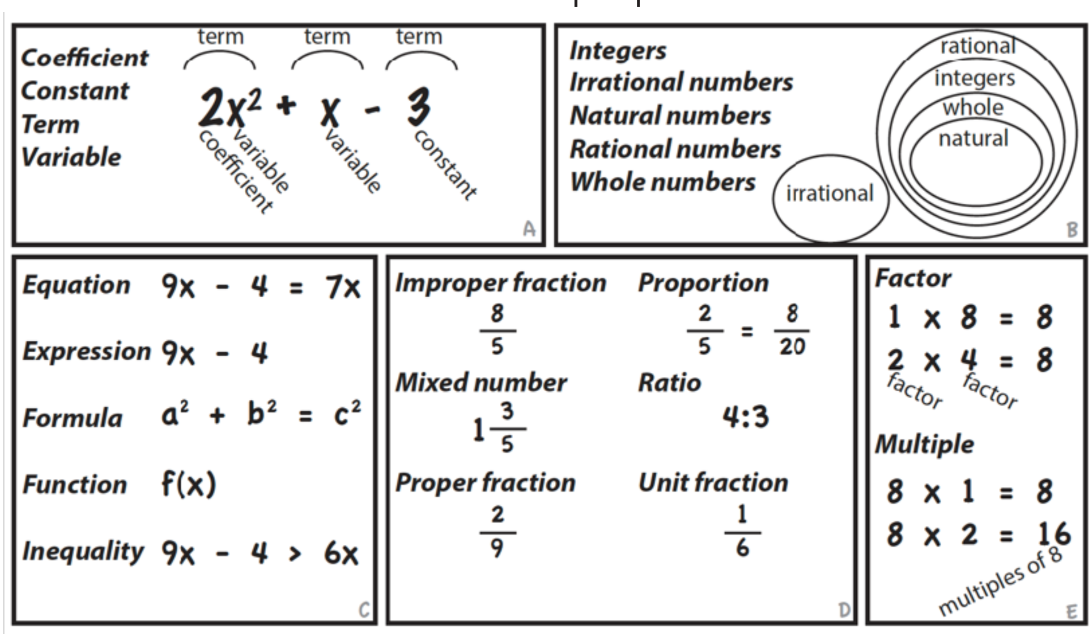


Figure 20. Examples of Mathematics Language

Mathematics Language

HOW TO TEACH MATHEMATICS LANGUAGE

Teachers should explicitly teach mathematics terms just as they would explicitly teach mathematics concepts to students. Additionally, teachers can pre-teach potentially challenging terminology and provide repeated exposures in multiple formats (e.g., orally, in writing). Visual tools supporting mathematics language development include Frayer models, mathematics journals, and word walls. See the figures below for images and descriptions of these tools.

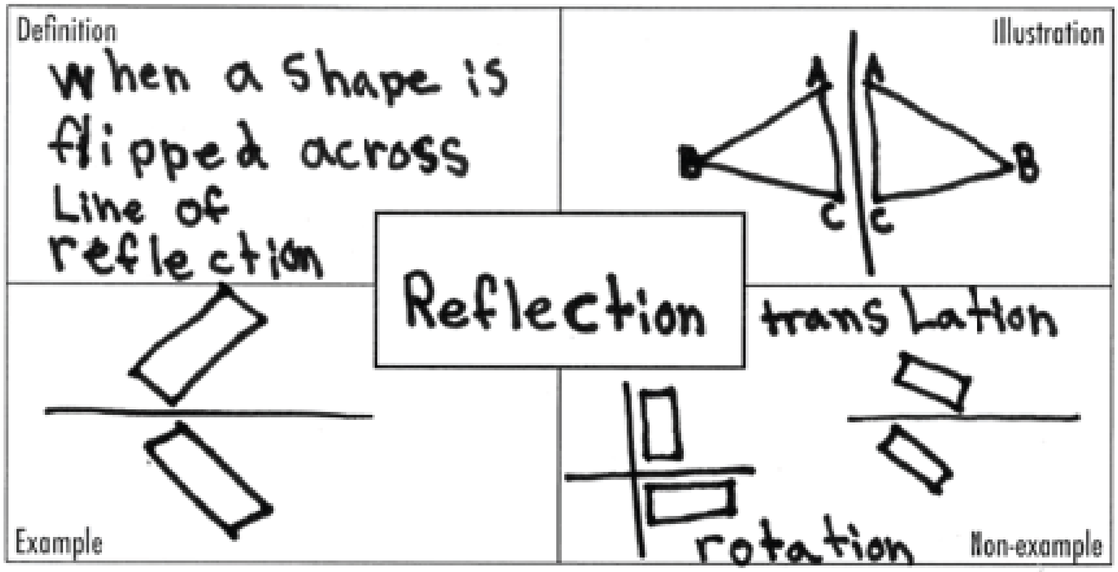


Figure 21. Example of Frayer Model A
Frayer model helps students break down a term into different components, including its definition, visualization, and examples versus non-examples.

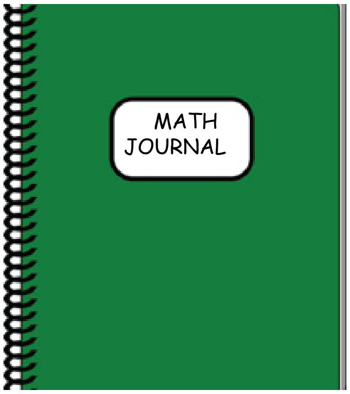


Figure 22. Example of Math Journal
Students can use mathematics journals to respond to mathematics writing prompts, such as describing their problem-solving process for a multi-step word problem.

Fluency-Building

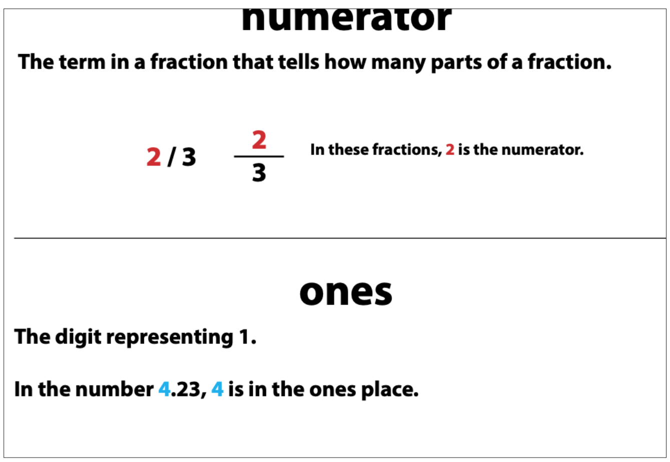


Figure 23. Example of a Word Wall
Teachers can document terminology on a word wall visible throughout the classroom. For each term, they can include a student-friendly definition and examples.

FLUENCY

Fluency-building activities provide opportunities for students to master mathematics facts (i.e., **fact fluency**) and other necessary mathematics knowledge (i.e., **procedural fluency**). **Fact fluency** includes recall of problems with single-digit addends, minuends, factors, and quotients. In addition to mathematics facts, it is recommended that middle-school students become fluent in determining equivalent and benchmark fractions, determining common denominators, and adding, subtracting, multiplying, or dividing positive and negative integers. Proficiency with these skills is called **procedural fluency.**

It is recommended that all teachers include daily brief fluency-building activities in mathematics class (Burns et al., 2010). Fluency practice is important for students with limited working memory and students with mathematics difficulty (MD). When students are fluent in essential mathematics facts and procedures, it frees up cognitive processing power, which can then be devoted to more complex mathematics content. For example, research demonstrates that fluency is associated with improved outcomes in algebra and word-problem solving (Fuchs et al., 2016).

Students who experience difficulties in mathematics often benefit from explicit instruction in strategies that help them learn math facts and perform procedures effectively. Teachers should provide these students with brief (i.e., 1 or 2 minutes) daily fluency-building practice.

Fluency-Building

FLUENCY-BUILDING ACTIVITIES

Many activities can support students' fact and procedural fluency. Students should be exposed to reasoning strategies to build fluency and conceptual understanding. See Figure 24 for examples of reasoning strategies for fact fluency in addition, subtraction, multiplication, and division below.

Addition:
- One More and Two More Than (Count On); 3 + 1 = 4
- Adding Zero; 5+ 0 = 5
- Doubles; 6 +6 = 12
- Combinations of 10; 6 + 4 = 10
- Making 10; 6 + 8 = (4 + 2) + 8 = 4 + 10 = 14
- Using 5 as an anchor; 7 + 6 = (2 + 5) + (1 + 5) =
  2 + 1 + 5 + 5 =
  3 + 10 = 13

Multiplication:
- Foundational Facts First; 2, 5, 10, 0, and 1
- Nines; 10 − 1
- Adding of Subtracting a Group; 6 x 4 = (5 x 4) + 4
- Doubling & Halving; 6 x 8 = (3 x 8) x 2
- Break Apart; 8 x 6 = (5 x 6) + (3 x 6)

Division:
- Think Multiplication; 36 ÷ 4 = ☐ is the same as
  4 x ☐☐ = 36
- Practice Near Division Facts; 50 ÷ 6

Subtraction:
- Think Addition; 13 − 8
  What plus 8 equals 13?
- Down under 10; 13 − 8
  (1) 13 − 10 = 3
  (2) 10 − 8 = 2
  (3) 3 + 2 = 5
- Take from 10; 13 − 8
  (1) 13 = 10 + 3
  (2) 10 − 8 = 2
  (3) 3 + 2 = 5

Figure 24. Examples of Reasoning Strategies for Fact Fluency

In addition to introducing the reasoning strategies above, students can practice fact and procedural fluency using various activities. Recommended examples include cover-copy-compare, taped problems, dice and card games, dominoes, fluency apps and digital games, and flash cards.

- Cover-copy-compare: Students cover up the correct answer, write their response, and then compare it to the correct answer.
- Taped problems: Students must solve the problem within a set time before the teacher, or audio-recorded tape reads the correct answer.
- Fluency game using dice, cards, or dominoes: Allow students to draw cards, roll dice, or pick dominoes and then add, subtract, multiply, or divide the numbers chosen. You can also use these tools to create fractions to practice fraction operations.

Word-Problem Instruction

Teaching Attack Strategies & Schemas

Many students require explicit instruction in solving word problems. It is important that teachers do not focus on teaching key words (Karp et al., 2019), but instead teach students how to use an **attack strategy** for working through word problems (Powell & Fuchs, 2018). High-quality attack strategies require students to read the problem, create a plan for solving the problem, execute the plan, and check the answer. Attack strategies should be flexible to address any word problem. See Figure 25 for some examples of attack strategies.

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers

UPS CHECK

Understand: Read the problem. Plan: Choose a strategy. Solve: Show all your work.

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with

RIDGES Read the problem. I know statement.

Draw a picture.

Goal statement.

Equation

Figure 25. Examples of Attack Strategies

As demonstrated above, many attack strategies are formatted as mnemonics. A mnemonic is a device designed to help students remember a strategy, concept, or series of steps. Many mnemonics include a series of letters in which each letter represents a step or component of an intervention. Mnemonics are evidence-based tools that have been shown to aid students in mastering mathematics concepts (Cuenca-Carlino et al., 2016; Freeman-Green et al., 2015). Mnemonics are particularly useful for students with working memory challenges.

Word-Problem Instruction

In addition to teaching students to use attack strategies, it is also important to introduce schemas (Jitendra et al., 2015). A schema refers to the structure of a word problem. There are three primary additive schemas: Total, Difference, and Change problems (Powell & Fuchs, 2018). There are also three multiplicative schemas: Equal Groups, Comparison, and Ratio/Proportion problems. See Figure 26 for depictions of each schema and Figure 27 for sample problems that fit into each category.

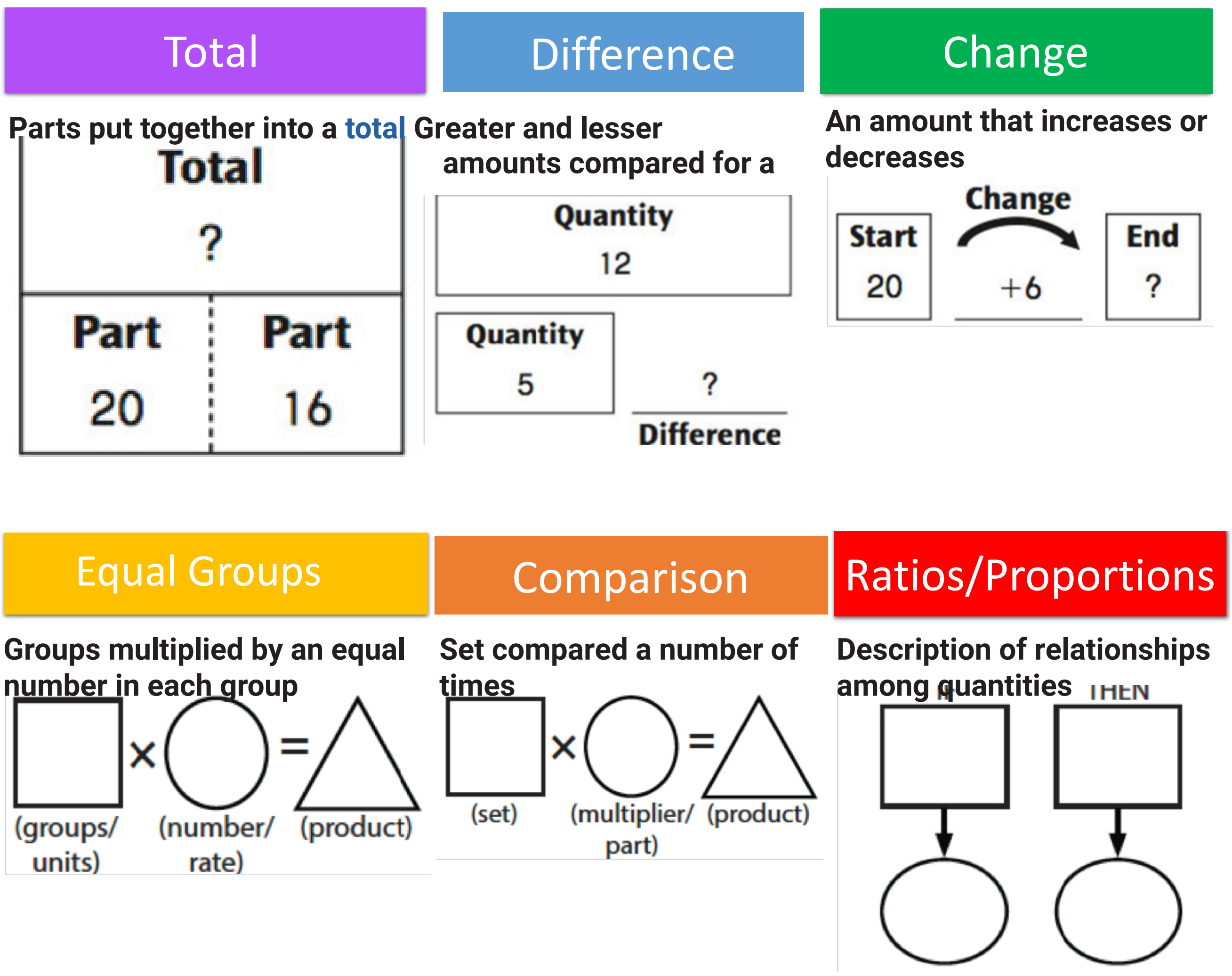


Figure 26. Depictions of Schema Types

Word-Problem Instruction

Total	Max baked 40 cookies and 75 brownies. How many baked goods did Max bake?
Difference	The Brazos River is 840 miles. The Red River is 1,360 miles. How much longer is the Red River?
Change	There were 23 passengers on the bus. Then, 13 more passengers boarded the bus. How many passengers are on the bus now?
Equal Groups	Mark has 2 boxes of crayons. There are 24 crayons in each box. How many crayons does Mark have?
Comparison	Jill picked 6 apples. Meg picked 2 times as many apples as Jill. How many apples did Meg pick?
Ratios/Proportions	There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slides of bread are in 5 loaves?

Figure 27. Sample word problems that fit into each schema category

Finally, see Figure 28 for an example of how a student can solve a word problem using an attack strategy plus their knowledge of schemas. This example is a multi-step problem with two schemas embedded: total and equal groups.

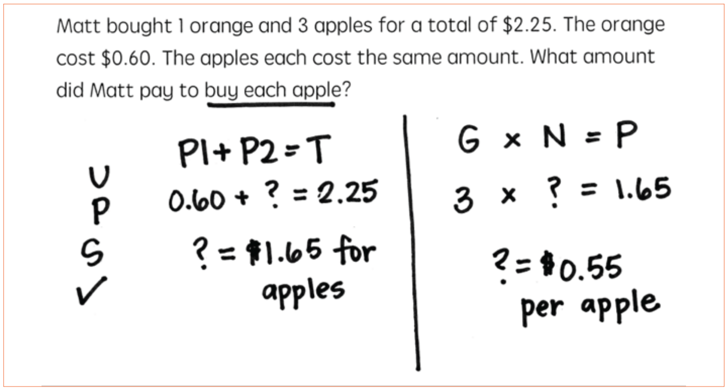


Figure 28. Example of Multi-Step Problem

Graphic Organizers

DEFINITION OF GRAPHIC ORGANIZERS

Graphic organizers are used to visually represent procedures or relationships among mathematics concepts. Graphic organizers can be useful in helping students to remember specific procedures. Alternatively, graphic organizers can be used to help students better understand relationships among mathematics concepts or learn mathematics vocabulary. Depending on the purpose of the graphic organizer, students can either be presented with a prepared graphic organizer, or students may benefit from drawing their own.

The efficacy of graphic organizers has been demonstrated in mathematics intervention research (Shin & Bryant, 2017; van Garderen, 2007). Additionally, the effectiveness of graphic organizers can be enhanced when combined with mnemonics and explicit instruction.

A common type of graphic organizer is called a Frayer model. In a Frayer model, students identify key characteristics of a term or concept, such as its definition, examples, and non-examples. This type of organizer can be helpful when introducing new mathematics vocabulary to students. See Figure 29 for a Frayer model graphic organizer.

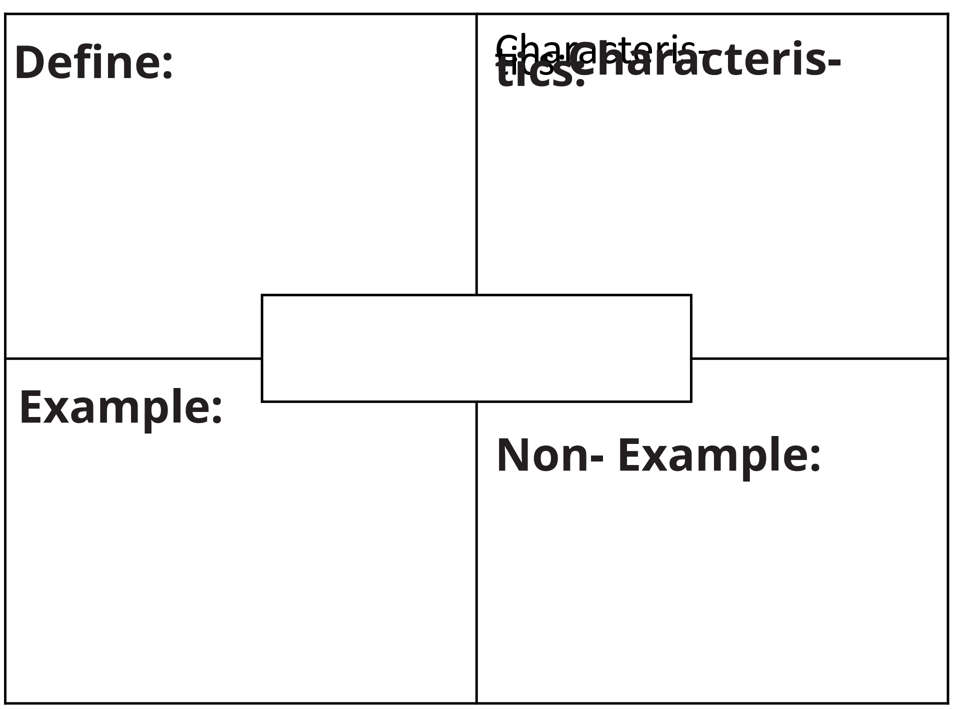


Figure 29. Example of Frayer Graphic Organizer

Graphic Organizers

Other types of graphic organizers include schematic diagrams (e.g., to visualize word problems), ten frames, and Venn diagrams. These organizers can be used for various purposes, including problem-solving, computation, and visualization of relationships across mathematics concepts. See Figure 30 below for examples of each of these graphic organizers.

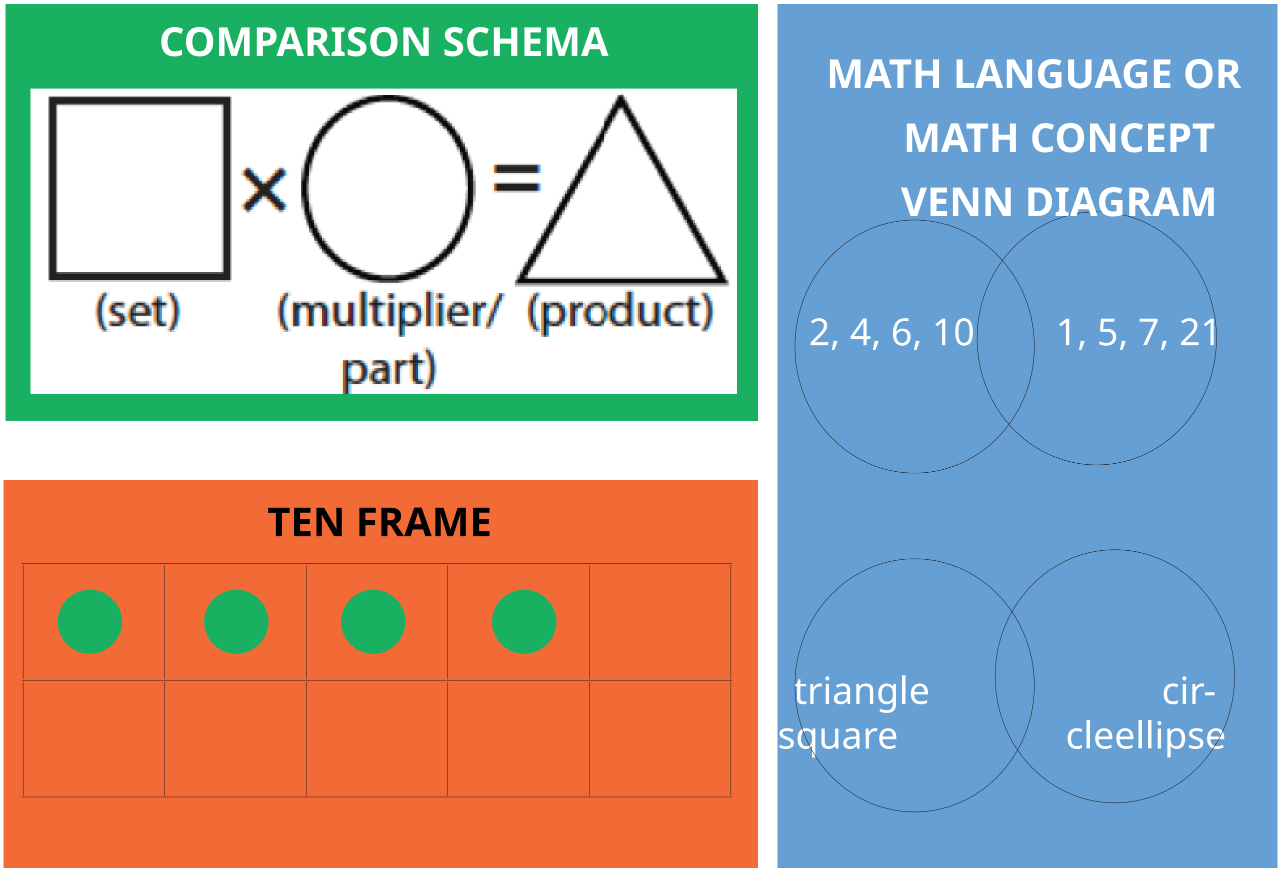


Figure 30. Examples of Various Graphic Organizers

Introduction to Intensification

WHAT IS AN INTENSIFICATION?

A key component of effective intervention intensification is working from a solid Tier 2 (i.e., targeted intervention) foundation. That is, before considering whether an intervention requires intensification, you should confirm a validated Tier 2 intervention program is in place and implemented with fidelity. Once you have determined a validated Tier 2 intervention is being implemented with fidelity and you identify a student who is not making adequate progress in their math learning, you can adapt or intensify the intervention. There are five ways to intensify math interventions:

1. **Implement With Greater Fidelity:** New interventions and strategies are only as good as how well they are implemented. You should implement your interventions and strategies with your students with fidelity. This may involve keeping checklists of the steps required to successfully implement the intervention. Implementing with fidelity ensures that the level of success of an intervention is not a factor in how the intervention is used but possibly a reflection of the appropriateness of the intervention or strategy itself.
2. **Embed Behavioral Supports:** While classroom management is an important task for every teacher, student self-regulation should be used and built upon to decrease nonproductive behavior. Embedding behavioral supports may be necessary because students experiencing math difficulty may struggle with paying attention and may not be focused to the extent of the teacher's preference. Utilizing behavioral support strategies will assist students with remaining on task and fully participating in the math intervention.
3. **Increase Dosage:** Sometimes students require more time working on an intervention. Many students experiencing difficulty with math need foundational math support. To assist these students with learning the foundations of math, teachers may need to develop strategies to increase the amount of math practice.
4. **Adapt Math Content:** Teachers can change the math content students receive. Students experiencing difficulty in math may require the content to be received in a specific sequence or with a focus on student strengths. Teachers can inject additional lessons to fill in gaps in math knowledge. Additionally, teachers can rearrange the scope and sequence or break it down into smaller steps.
5. **Teach for Transfer:** Explicitly teaching for transfer ensures students take examples worked in class and utilize the knowledge in more complex problems. Teachers should teach beyond memorizing concepts and procedures and ensure students have a rich understanding of math to apply the content knowledge to other scenarios.

Fidelity

TIPS FOR SELECTING A STRATEGY

Before choosing an intensification strategy(ies), we recommend reading through this guide to determine which strategies you already have in place and which would be feasible to implement in your current setting.

We recommend starting with one intensification strategy at a time, using data to determine whether there are changes in student learning. Once you choose an intensification strategy, consistently implement the strategy to determine the results.

When Implementing: Track your students' progress for several weeks. If the intensification strategy is implemented consistently and a student has yet to reach their goal, select a second intensification strategy to try. Remember, every student is different, and only some strategies will work for some students. It is important to stay flexible, and collecting data to make informed decisions is critical!

IMPLEMENT WITH GREATER FIDELITY

The purpose of implementing an intervention with fidelity is to ensure that the intervention is carried out as designed. Some interventions may allow for greater flexibility, but all interventions include key components that have been researched and found to impact student outcomes positively. If these key components are not implemented or carried out as designed, teachers may not see the promised benefits of the intervention. Examples of key elements may include the sequence of activities, mastery criteria, or the delivery of the intervention.

Some ways to monitor fidelity:

- Creating a checklist of key components helps ensure high fidelity to an intervention. This list may be provided in the intervention's manual, or it may need to be created. If a checklist needs to be made, the original research team can be contacted to help, or you can create one by identifying the key components of the intervention.
- Invite another teacher or instructional coach to observe an intervention session. Ask her to complete the checklist and provide honest feedback about which parts of the intervention you are implementing well and which parts you are not including.
- After class, debrief with your observer about how you can improve your intervention implementation. If a colleague is unavailable, video record yourself teaching and review the fidelity checklist afterward.
- Keep the checklist on your desk or in sight when implementing the intervention. Be sure to ask questions and be receptive to critical feedback.

Behavioral Supports

EMBED BEHAVIORAL SUPPORTS

The purpose of embedding behavioral supports is to decrease nonproductive behavior and assist students with remaining on task during math intervention. According to Fuchs, Fuchs, and Malone (2017), embedding behavior supports may be a necessary adaptation. This is because many students need help with paying attention, including students experiencing math difficulty, and may need to be more focused on the math lesson to the extent needed to learn and be successful.

Reinforce the behavior you want to see by defining the behavior and then finding a reinforcer the student(s) wants to work towards achieving. For example, if a student(s) has struggled to stay focused during math, the behavior you want to see is increased focus. What does "focus" look like and sound like? Focus needs to be operationally defined so the teacher knows what to look for and the student knows what is expected. An expected behavior should be defined and communicated with the student(s), and a reinforcer should be decided. There is no such thing as a universal reinforcer. For this reason, it is important to find out what the student(s) finds reinforcing. The reinforcement can be tangible (e.g., a prize) or non-tangible (e.g., extra time to talk with peers or use electronics). See Figure 31 for example strate-

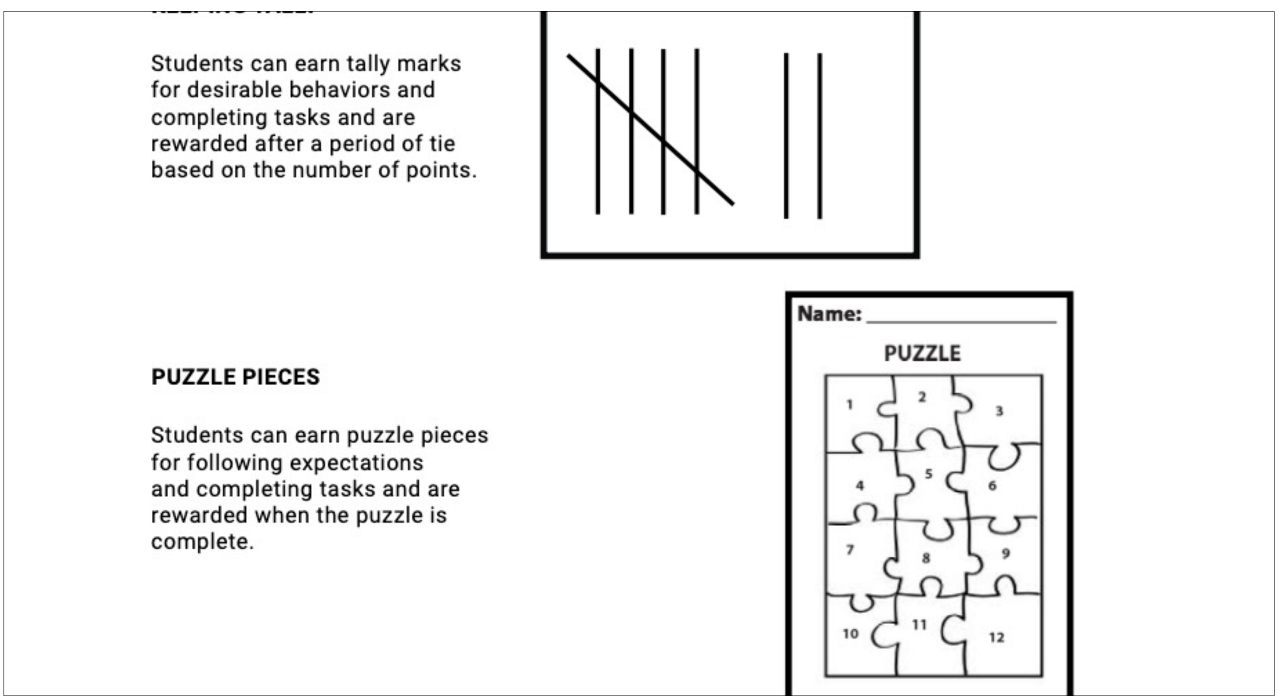


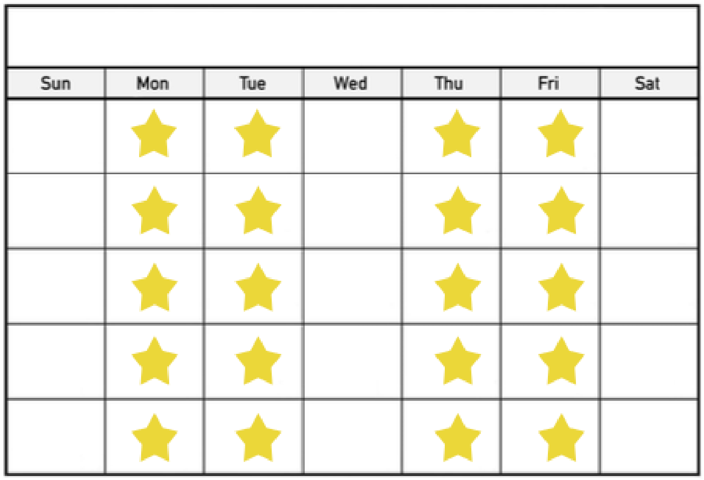
Figure 31. Examples of Reinforcement Strategies

Increased Dosage

HOW DO I INCREASE DOSAGE?

Dosage is the amount of intervention a student receives, which can include the number of opportunities to respond within a session, the number of sessions, the number of days per week, or the number of weeks overall.

The purpose of intensifying dosage is to determine the appropriate combination of intervention frequency (i.e., how often) and duration (i.e., for how long) for a particular student. For example, if schedules permit, you can with meet students more frequently for intervention. For example, instead of only meeting twice a week, you can meet four times a week.



WHERE AND HOW DOES THIS INTENSIFICATION TAKE PLACE?

Increased dosage often (but not always) means increased time. Therefore, when trying to intensify the frequency or the duration of an intervention, maximize resources in the classroom by utilizing peers, paraprofessionals, related service providers, or configurations (e.g., station teaching) that allow for increased engagement with the intervention. If you are using a validated intervention, consult the manual on the recommended dosage and note whether the authors recommend intensification. If this information is unavailable or you are not currently using a validated intervention, consider the following procedures to intensify the intervention dosage.

What do I increase?	What might it look like?
Intervention session length	15 mins twice per week → 30 mins twice per week
Number of daily sessions	15 mins in the am → 15 mins in the am and pm
Number of weekly sessions	Twice per week → 4 times per week

Adaptations to Math Content

ADAPTATIONS

The purpose of adapting the math content is to ensure that students can access the core math concepts. This can include adapting how the material is presented. Teachers can focus on using precise language, scaffolding activities, or teaching strategies such as word-problem attack strategies. Some students may require more intensive support, and teachers can adapt the scope and sequence of units to ensure that critical content is targeted and builds upon students' understandings. If you are modifying the curriculum, it is important that the IEP team together makes this determination. Adjusting teacher practices may be helpful for whole-class intervention or small groups. Adjusting the scope and sequence for a student's needs will be feasible for intensive interventions, either during small group or one-on-one tutoring. See below for whole-class adaptations.

WHOLE-CLASS ADAPTATIONS

ENGAGE STUDENTS IN DISCOURSE

"Tell me how you solved this problem." "What were you thinking about when you regrouped?" "How would you teach this problem to another student?"

"Describe the word problem in 10

PROVIDE WORKED EXAMPLES

"Talk through this problem with me."

$$\begin{array}{r} 405 \\ + \\ \hline 411 \end{array} \qquad \begin{array}{r} 405 \\ + \\ \hline 421 \end{array}$$

TEACH PROBLEM-SOLVING STRATEGIES

1. Don't tie key words to operations
2. Have an attack strategy
3. Teach word-problem schemas

Figure 32. Example of Whole-Class Adaptation

Adaptations to Math Content

ADDITIONAL WHOLE-CLASS ADAPTATIONS

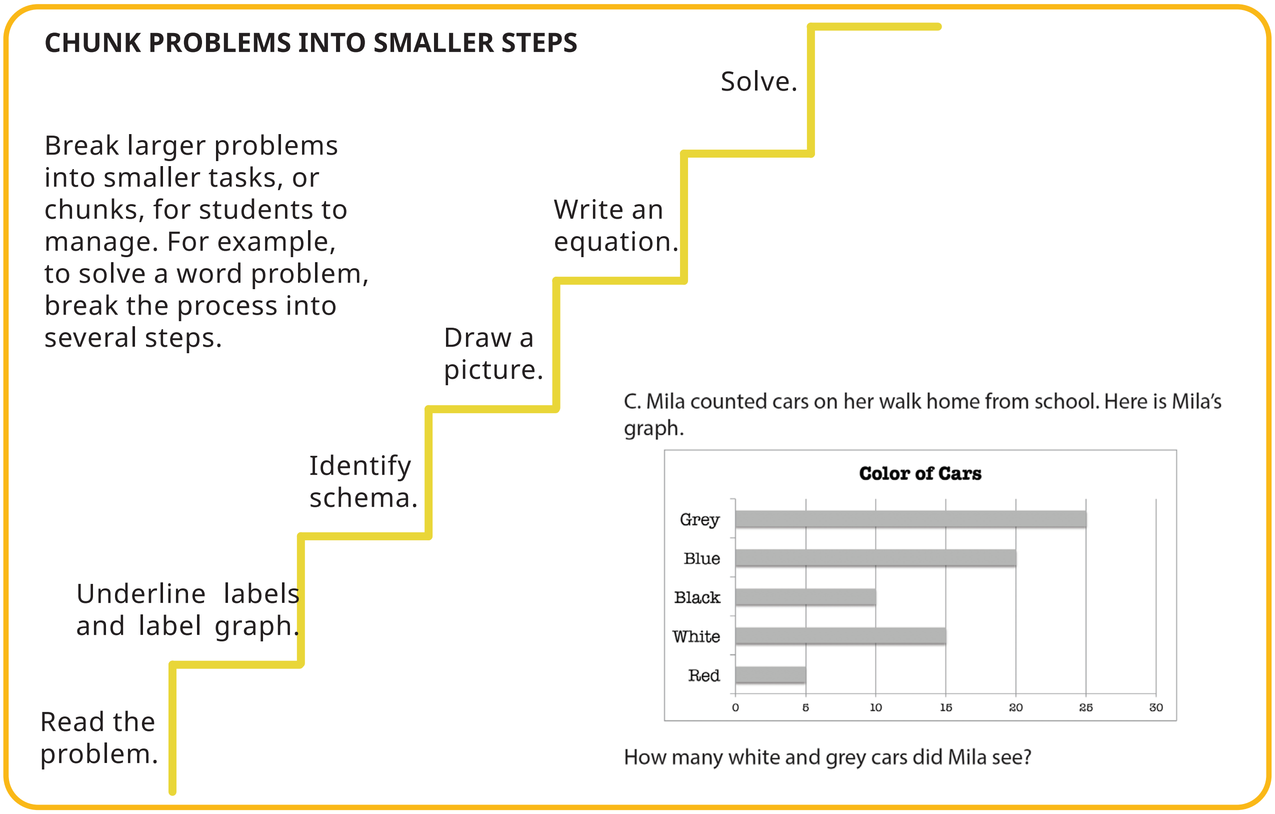


Figure 33. Example of Whole-Class Adaptation

INDIVIDUALIZED ADAPTATIONS

Some students may require individualized adjustments to master critical content. Teachers, working with interventionists or researchers, can identify which skills the student has mastered, where the student needs to be by the end of the term or year, and what is essential for the student to master those end-of-year goals. Modifying a curriculum requires an in-depth understanding of math standards, common math progressions, and the individual student's IEP goals. Therefore, we recommend that teachers only alter the scope of sequence as a last resort and always with the knowledge and support of administration and interventionists.

Teaching for Transfer

WHAT IS TEACHING TO TRANSFER?

Transfer is a student's ability to recognize and apply features of previous learning to a novel problem or context. The purpose of explicitly teaching for transfer is to intensify the conceptual connections students are making between math content. Examining the scope and sequence across multiple instructional units can reveal opportunities for students to draw on prior knowledge and apply similar skills or concepts to new situations or with more complex content (e.g., principles for solving one-step equations applied to solving two-step equations).

HOW DO I EXPLICITLY TEACH TRANSFER?

1. Explicitly name the similar characteristics of the old and new problems or contexts. For example, you could pose these problems: **Morgan spent $42 for shoes. This was $14 less than twice what they spent for a shirt. How much was the shirt? Pat wanted to organize their button collection. There were 42 buttons in the collection. 14 buttons were lost from a container that had twice as many than what was originally in the container. How many buttons were in the container?**

2. Ask students to identify the relationships in these two problems—how are they similar? How can solving the first problem help them solve the second problem?
3. To extend this transfer, ask students to represent either problem in abstract notation.
4. Continue drawing students' attention to how the skills and relationships in one problem can support their understanding or approach to another problem in a unique context.

We hope you found this manual to be a valuable resource when implementing DBI in your mathematics classroom. Thank you for participating in STAIR 2.0! For additional support, check out the Resources section

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Resources

STAIR Video Resources

Topic	Link
Defining DBI	https://www.youtube.com/ watch?v=1gmVAWea-Svs&t=177s
National Center On Intensive Intervention	https://intensiveintervention.org
NCII's DBI Framework	https://intensiveintervention.org/sites/default/files/ DBI_Framework.pdf
STAIR 2.0 Overview	https://www.smu.edu/Simmons/Research/Research-in-Mathematics-Education/Explore/ STAIRR
Implementing Mathematics DBI During COVID-19	https://intensiveintervention.org/resource/ webinar-series-implementing-dbi-mathematics-during-covid-19-restrictions

RESOURCES FOR ASSESSMENT IN THE VIRTUAL ENVIRONMENT

Topic	Link
Blended learning and assessments	https://www.istation.com/
Flexible delivery options for assessments	https://www.nwea.org/
Using iReady assessments in the virtual environment	https://www.curriculumassociates.com/ products/i-ready/how-i-ready-supports-teachers-leaders-2020-2021
Covid-19 response resource	https://www.nciea.org/current-initiatives/covid-19-response-resources
Project STAIR Youtube Channel	https://youtube.com/@ project-stair9458?si=Wokz4C7JtAT_rg7rUCE2puwDtUSNX-FONIOhmYmvA

Project STAIR

Teacher Reference Manual

Supporting Teaching of Algebra Individual Readiness 2.0

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College of Education, University of Missouri

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